

8th grade

Orange Belt Training: Functions Discipline

Order of Mastery: Graphing

1. Input/Output (F1)
2. Intercepts (F2)
3. $y=mx+b$ (F3)
5. Writing linear equations (F4)
6. Comparing linear equations (EE5)

Welcome to the Orange Belt – Graphing! That’s a pretty general topic. You’ve probably graphed before. You’ve been hearing about it since 6th grade. But this belt deals with all the ins and outs of what linear equations are and what a function can do for you. Ever heard of an algorithm? It’s basically a step-by-step procedure for calculations. You have an input and an output. Sound like a function? It’s what computers use to process data. Siri had to come from somewhere! But before we invent Siri, we need to master graphing.

Good Luck Grasshopper.

Important Terms:

X-intercept	Function Form	Point-Slope Form
Y-intercept	Standard Form	Slope-Intercept Form

Standards Included:

8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.¹

8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear

8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .

1. Input/Output

Remember that for something to be a function, for every input there is only one output. If you enter $4 + 6$ on a calculator you should get 10 every time. If we got something different each time man would that be confusing! Let's use our knowledge of functions to take an equation and represent it on a graph.

Steps on How to Graph

Example: Graph $2x + y = 4$

1. First make an X and Y table

X	Y

2. Pick a number for X: 0 or 1 is usually easiest, then plug that number into the equation and solve for y:

$$\begin{aligned}\text{Plug in 0 for x: } & 2(0) + y = 4 \\ & y = 4\end{aligned}$$

So when x is 0, y is 4. Put that in your chart -->

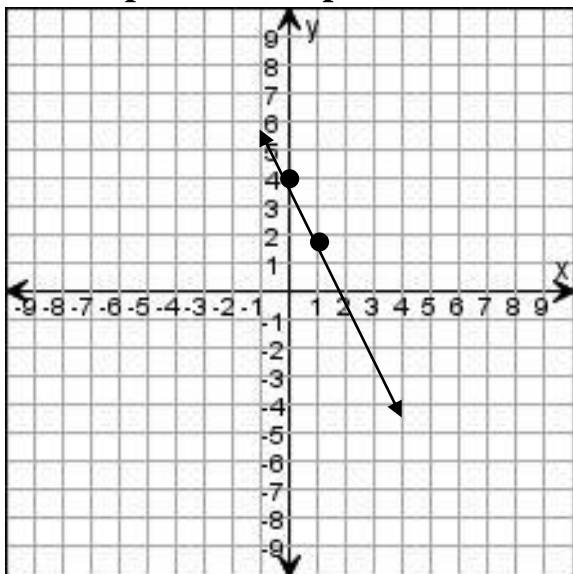
X	Y
0	4

3. Now pick another number for x and do the same thing

$$\begin{aligned}\text{Plug in 1 for x: } & 2(1) + y = 4 \\ & 2 + y = 4 \\ & -2 \quad -2 \\ & Y = 2\end{aligned}$$

X	Y
0	4
1	2

4. Now plot the two points and connect the dots! You have yourself a line!



5. To check your answer, just plug in one more value for x. If all your points line up, then you have the right line!

$$\begin{aligned}\text{Try } x = 2 & 2(2) + y = 4 \\ & 4 + y = 4 \\ & y = 0\end{aligned}$$

Does (2,0) line up? Yes!!!

Why X and Y?

Graphing Linear Equations

Checking solutions of linear equations:

Is the ordered pair (1,2) a solution of $x + 2y = 5$?

$(1) + 2(2) = 5$ Substitute 1 for x and 2 for y

$1 + 4 = 5$

Simplify

$5 = 5$

True statement. So yes, (1,2) is a solution of the equation $x + 2y = 5$

Determine whether the ordered pair is a solution of the equation: (YOU MUST SHOW ALL WORK)

1. $x - y = -7$ (-3, 4)

2. $x + y = 10$ (2, -12)

3. $6y - 3x = -9$ (1, -1)

Rewrite the equation in function form: (Solve for y)

4. $-3x + 5y = -15$

5. $x + 4y = -4$

6. $4x - \frac{1}{2}y = 8$

7. $\frac{1}{3}x + \frac{2}{5}y = 20$

Find three ordered pairs that are solutions of the equation:

8. $5x + 2y = 10$

9. $y - 3x = 9$

10. $-5x - 3y = 12$

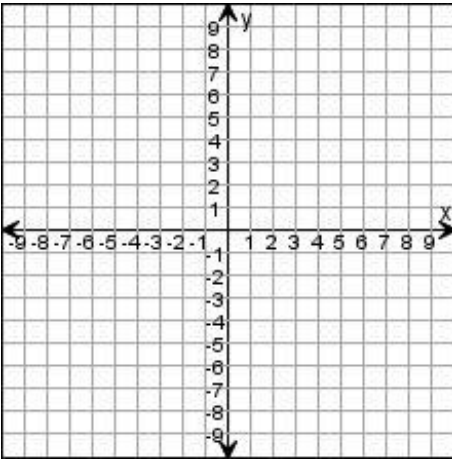
<u>X</u>	<u>Y</u>

<u>X</u>	<u>Y</u>

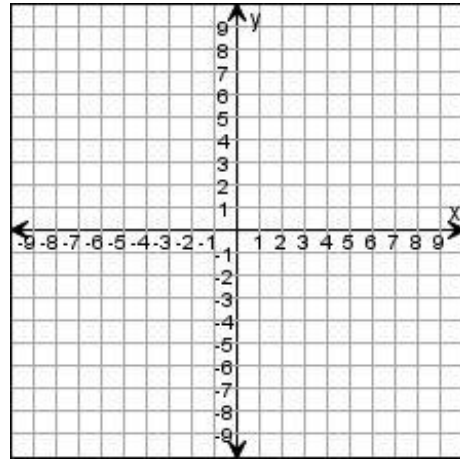
<u>X</u>	<u>Y</u>

Use a table of values to graph the following equations: (YOU MUST SHOW TABLE FOR CREDIT)

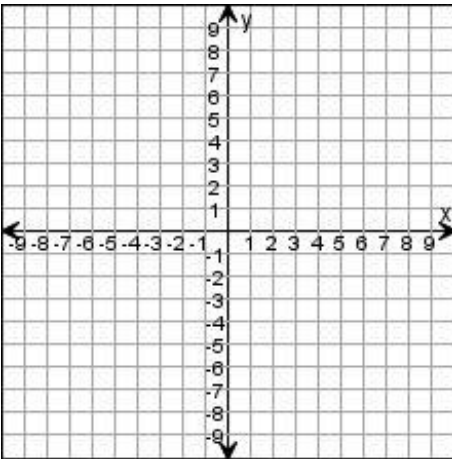
11. $2x + y = 3$



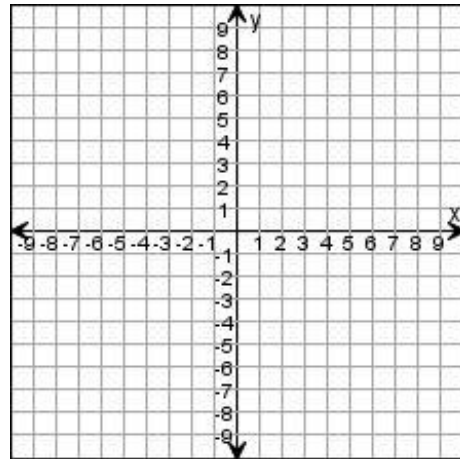
12. $y - 4x = -1$



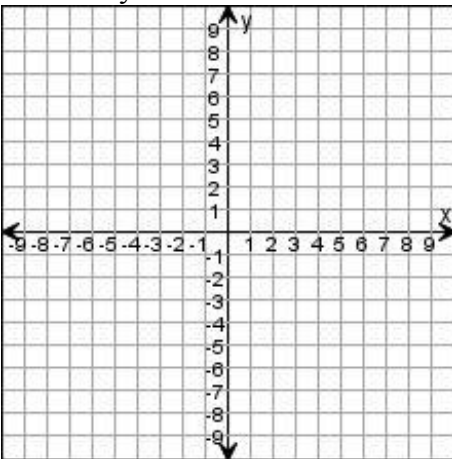
13. $5x + 5y = 25$



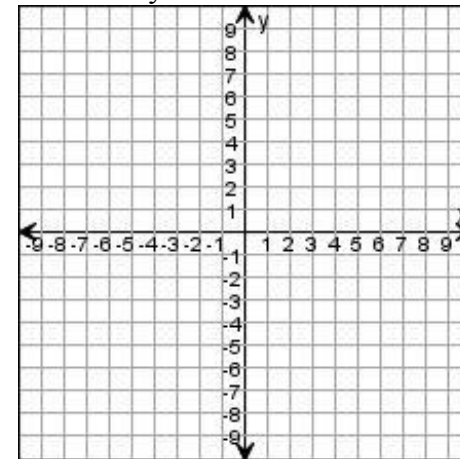
14. $y - 5x = -2$



15. $x + 4y = 48$



16. $-x + 2y = 6$

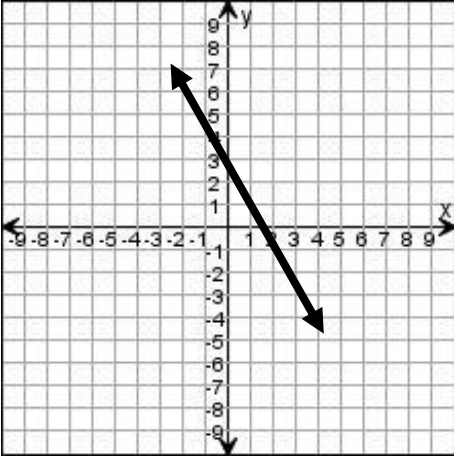


ANSWERS

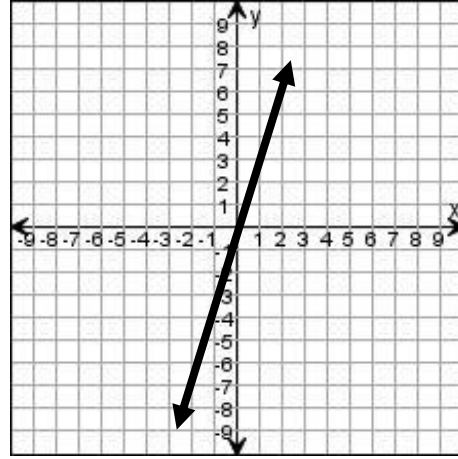
1. YES 2. NO 3. YES 4. $y = 3/5x - 3$ 5. $y = -1/4x - 1$ 6. $y = 8x - 16$ 7. $y = -5/6x + 50$

8. (0,5) (1, 2 1/2) (2, 0) 9. (0,9) (1, 12) (2,15) 10. (0,-4) (3,-9) (6,-14)

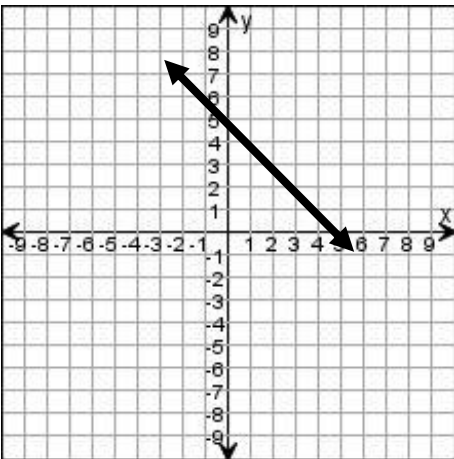
11. $2x + y = 3$



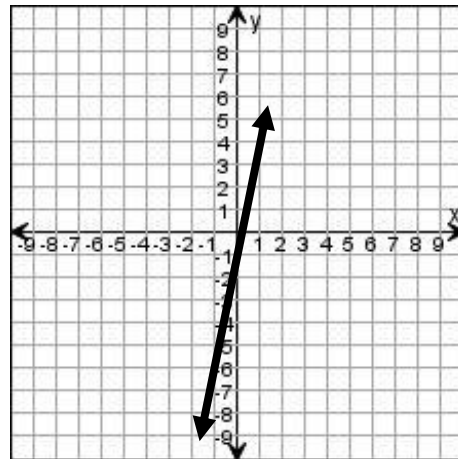
12. $y - 4x = -1$



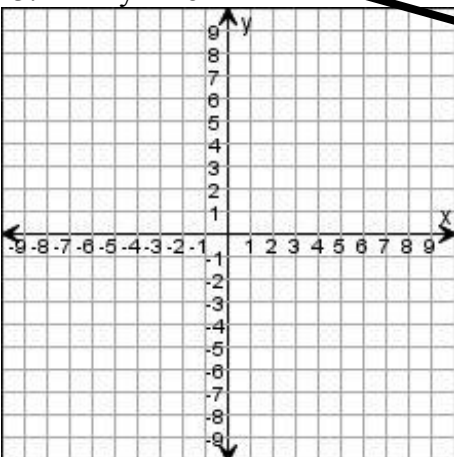
13. $5x + 5y = 25$



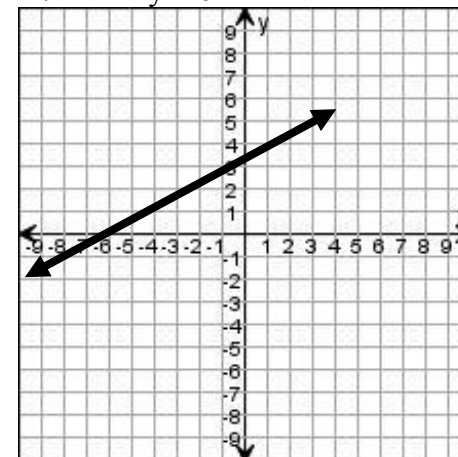
14. $y - 5x = -2$



23. $x + 4y = 48$



24. $-x + 2y = 6$



Build your own graph

On a SEPARATE PIECE OF GRAPHING PAPER, you'll need to make a coordinate plane for each one of these lines. Then graph them on your brand new graphs you made. Make sure you use the boxes provided on the graph paper (that's why they're there) SHOW ALL YOUR WORK on this piece of paper underneath the equation.

1. $y = -2x + 2$

2. $y = -3x - 3$

3. $y = -x + 10$

4. $y = x$

5. $y = 2x$

6. $y = 3x$

7. $y = -2x$

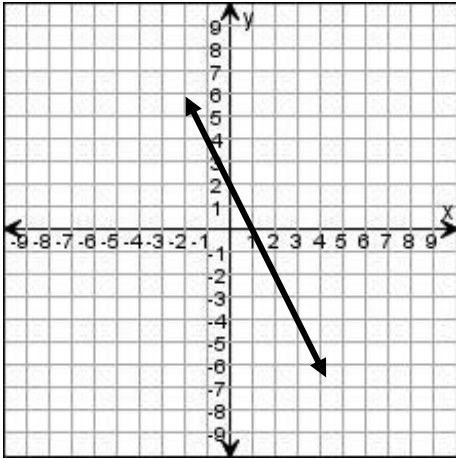
8. $x + y = 10$

9. $2x + y = 2$

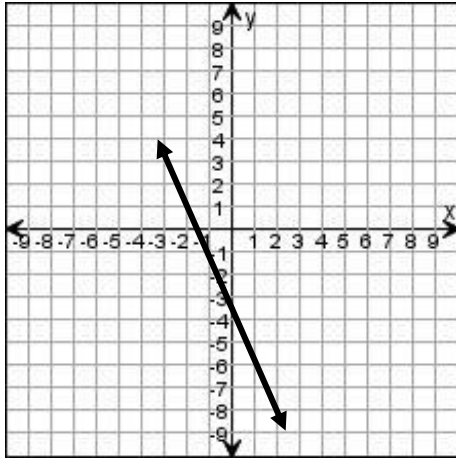
Look at your graphs. Do any of these look the same? If there are two that look EXACTLY alike, write down the number of the questions that matched right here: ___ matched ___ matched ___

ANSWERS

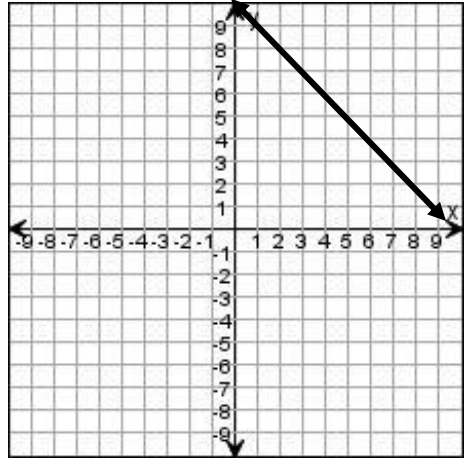
1.



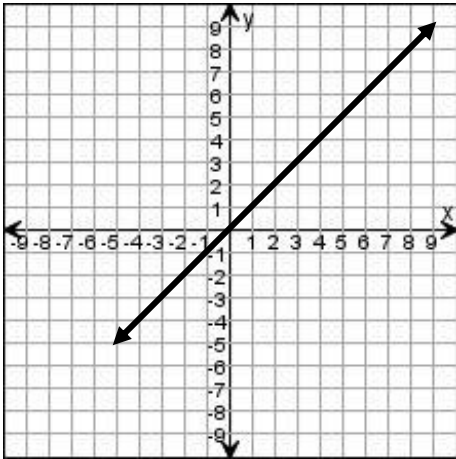
2.



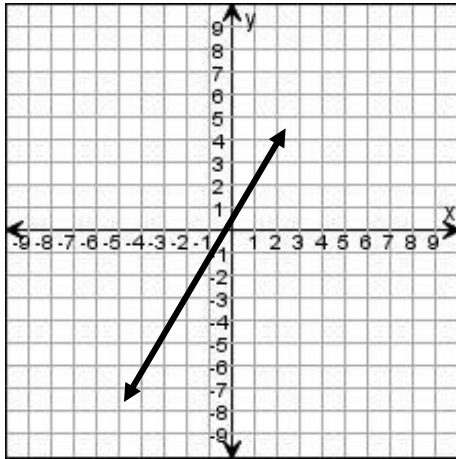
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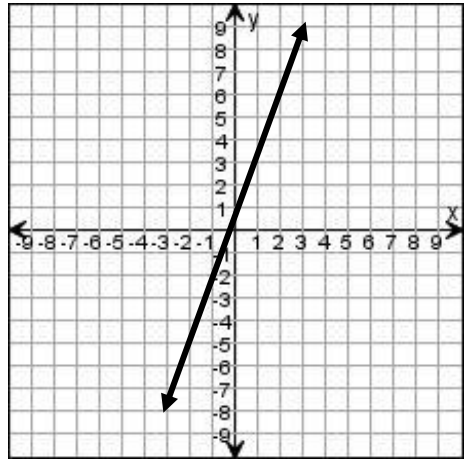
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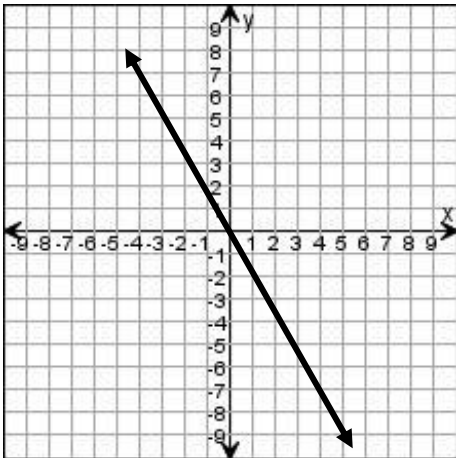
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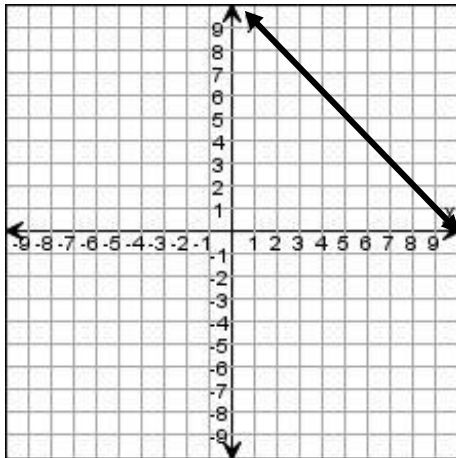
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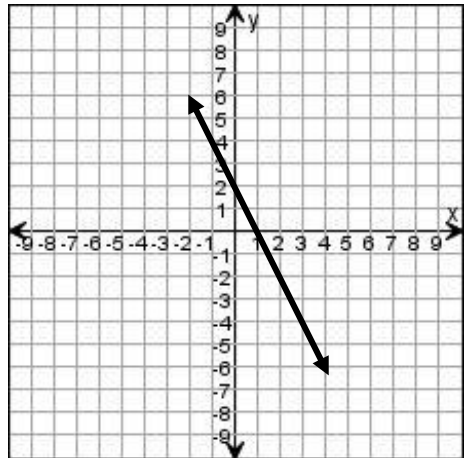
7.



8.



9.



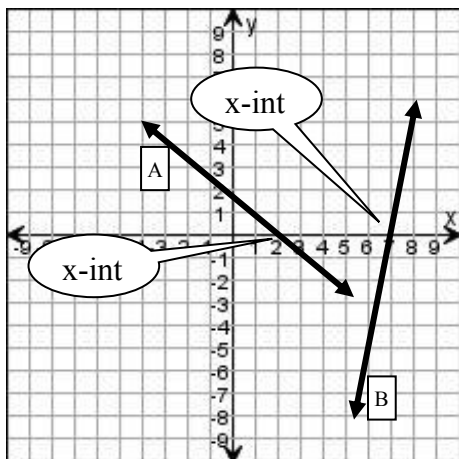
3 AND 8 MATCH 1 AND 9 MATCH

2. What about those intercepts?

The intercept on a graph is very easy to see, and can be very useful in graphing. It's especially good for checking your work and it's the easiest way to plot two points on a line.

There is an x intercept and a y intercept.

X-intercept: The point on the line where it's "intercepted" by the x-axis.
In other words, it's the point where the line crosses the x-axis.

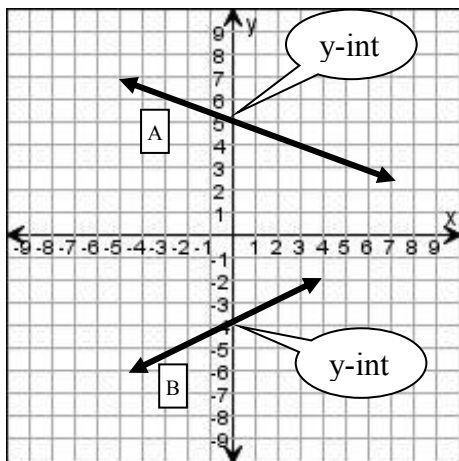


In line A the x-intercept is at the point (2,0)

In line B the x-intercept is at the point (7,0)

Notice that the y-value for both x-intercepts is 0. That's because the x-intercept is always when $y=0$.

Y-intercept: The point on the line where it's "intercepted" by the y-axis.
In other words, it's the point where the line crosses the y-axis



In line A the y-intercept is at (0,5)

In line B the y-intercept is at (0,-4)

There's another pattern here. The x-value is always zero for the y-intercept. So the y-intercept is where $x=0$

It's easy to spot the intercepts when looking at a graph. But you can also figure them out by looking at the equation of the graph. To find the x-intercept of a linear equation, just plug in 0 for y. To find the y-intercept of a linear equation, just plug in 0 for x.

Example: Find the x and y intercepts in the equation: $2x + 3y = 6$

$$\begin{aligned} \text{X-intercept: when } y=0 \quad 2x + 3(0) &= 6 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

So the x-intercept is at (3, 0)

$$\begin{aligned} \text{Y-intercept: when } x=0 \quad 2(0) + 3y &= 6 \\ 3y &= 6 \\ y &= 2 \end{aligned}$$

So the y-intercept is at (0, 2)

INTERCEPTING THE INTERCEPTS

Finding x-intercepts: Find the x-intercept of the line

1. $x - 2y = 4$

2. $x + 4y = -2$

3. $2x - 3y = 6$

4. $-x - 5y = 12$

5. $\frac{1}{2}x + 3y + 2 = 0$

6. $\frac{2}{3}x - y = 9$

Finding y-intercepts: Find the y-intercept of the line

7. $y = 4x - 2$

8. $y = -3x + 7$

9. $2x - 17y = -51$

10. $\frac{1}{2}y = \frac{1}{2}x + \frac{1}{2}$

11. $-y + 2x = \frac{3}{4}$

On a separate sheet of graph paper, graph the line that has the given intercepts:

12. x-intercept: -2
y-intercept: 5

13. x-intercept: 4
y-intercept: 6

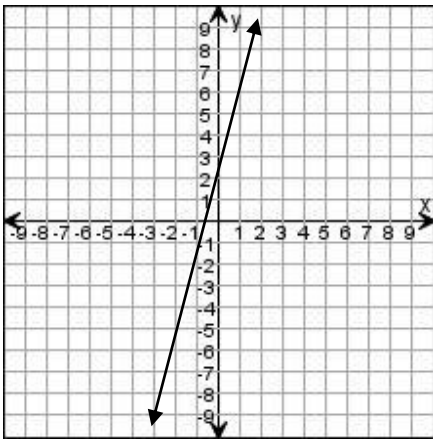
14. x-intercept: -3
there is no y-intercept

Match the equation with its graph. (place the number of the appropriate graph next to the equation.)

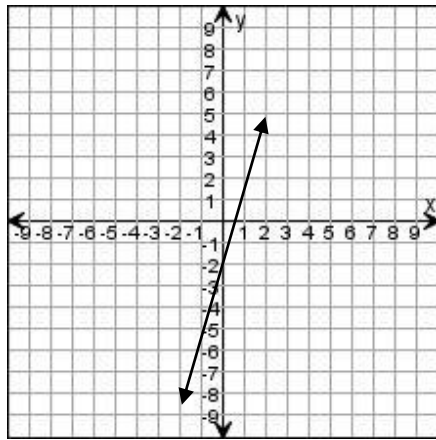
15. $y = 4x - 2$

16. $y = 4x + 2$

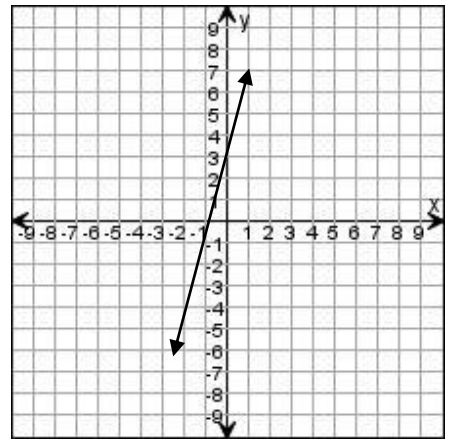
17. $y = 4x + 3$



1



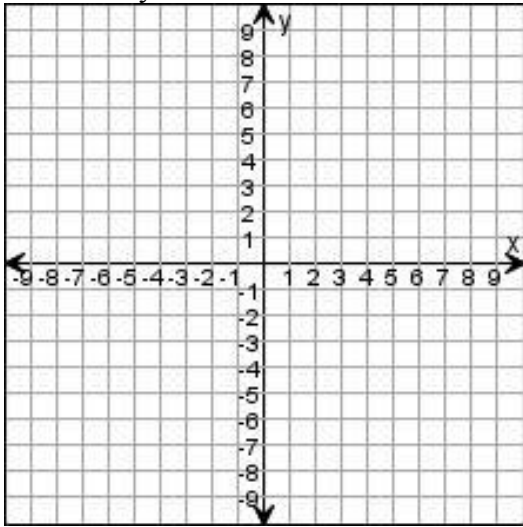
2



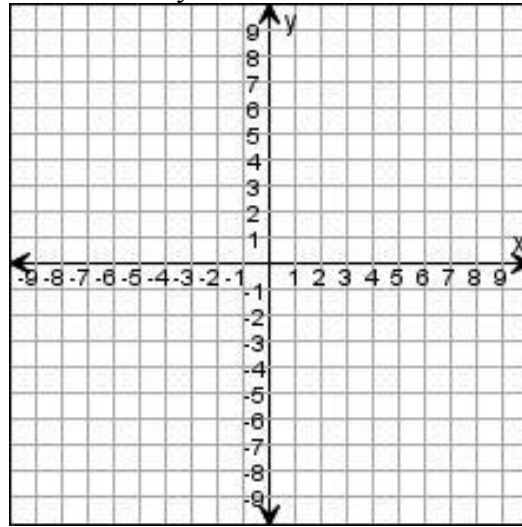
3

Let's perfect those graphing skills you genius you! Graph each equation (Label the points where each line crosses the x and y axes:

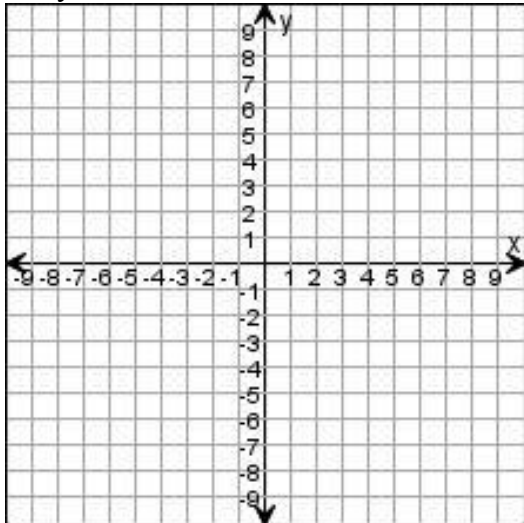
18. $-6x - 4y = 42$



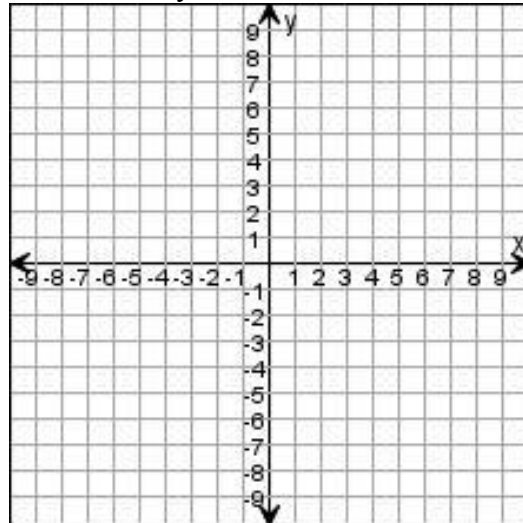
19. $9x - 4y = 54$



20. $y = 2 - x$



21. $2x - 17y = -51$

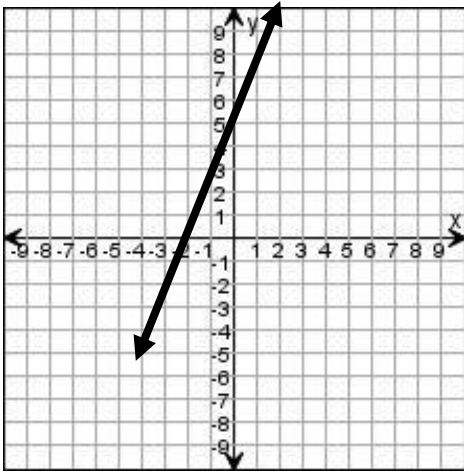


ANSWERS

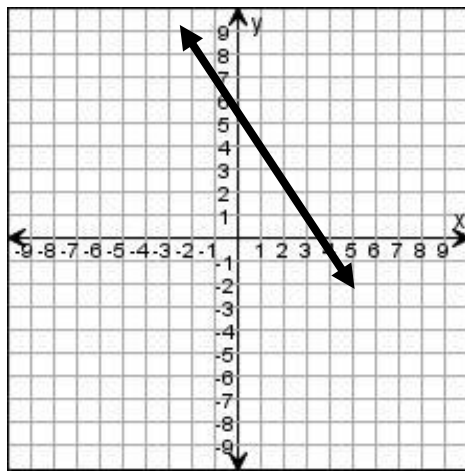
1. (4,0) 2. (-2, 0) 3. (3, 0) 4. (-12, 0) 5. (-4, 0) 6. (13.5, 0)

7. (0, -2) 8. (0, 7) 9. (0, 3) 10. (0, 1) 11. (0, -3/4)

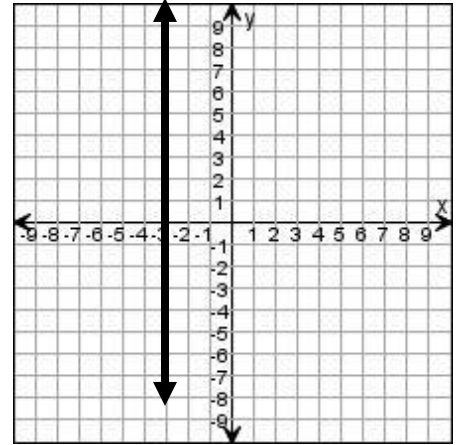
12.



13.

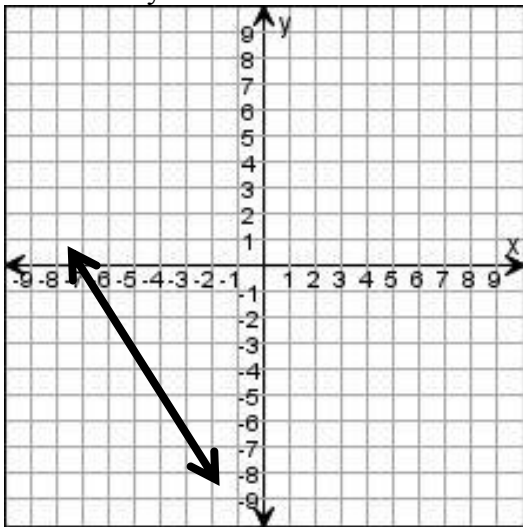


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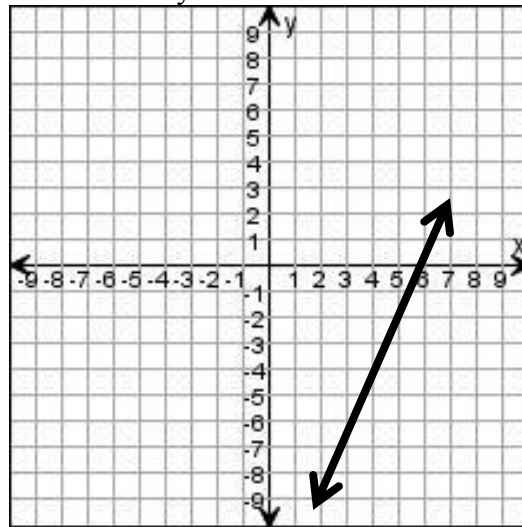


15. 2 16. 1 17. 3

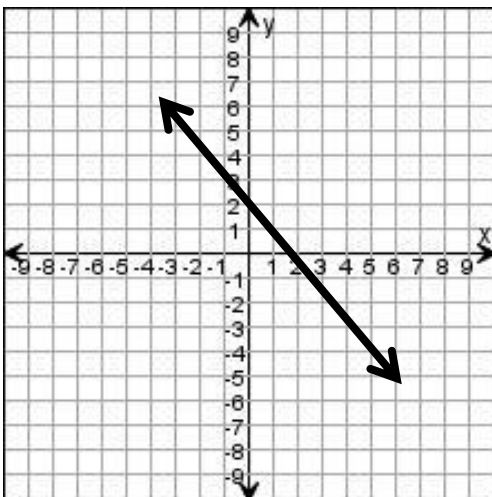
18. $-6x - 4y = 42$



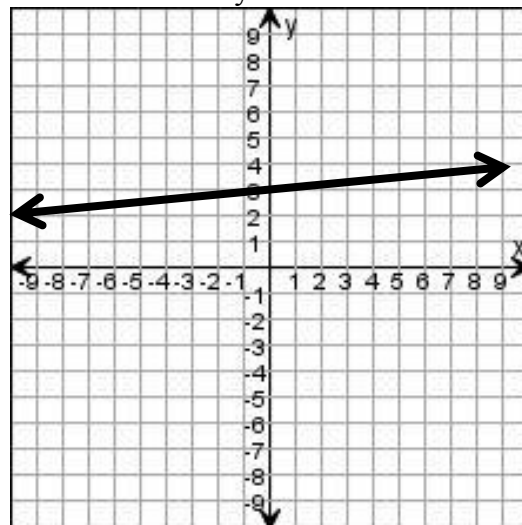
19. $9x - 4y = 54$



20. $y = 2 - x$



21. $2x - 17y = -51$



3. $y = mx + b$

Another way to graph: The Slope-Intercept way

Slope-Intercept form: $y = mx + b$ where m is the slope and b is the y-intercept

\updownarrow \updownarrow
slope y-intercept

Example:

Find the slope and y-intercept of the following equation:

$$2x + y = -3$$

Start by solving for y:

$$\begin{array}{r} 2x + y = -3 \\ -2x \quad -2x \\ \hline \end{array}$$

Subtract 2x from both sides

$$y = -3 - 2x$$

Put into slope-intercept form

$$y = -2x - 3$$

The slope is -2
The y-intercept is -3

\updownarrow \updownarrow
Slope y-intercept

Rewrite the equations in slope intercept form:

1. $3x + y = -1$

2. $10x - 5y = 50$

3. $y - 4x = 9$

4. $3x - 6y = 18$

Find the slope and y-intercept of the following equations:

5. $y = 3x - 7$

6. $y = 2x - 9$

7. $y = -2$

8. $y - 9x = 0$

9. $12x + 4y = 24$

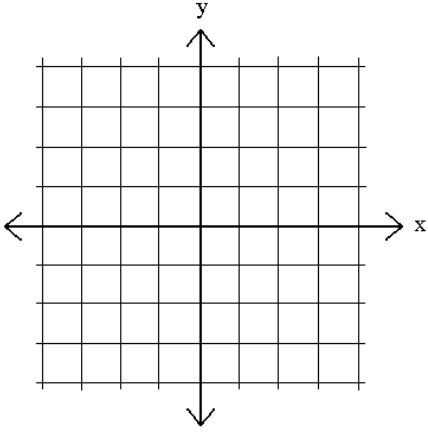
10. $3x + 4y = 16$

11. $7y - 14x = 28$

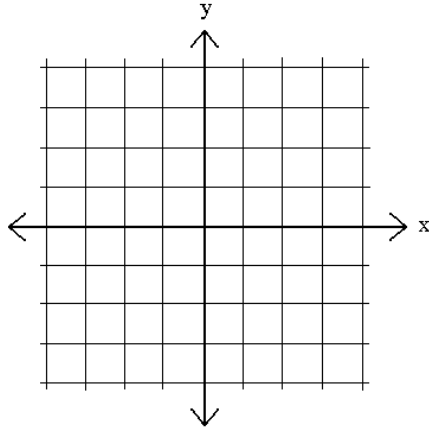
12. $-y = 2x + 3$

Graph the following equations:

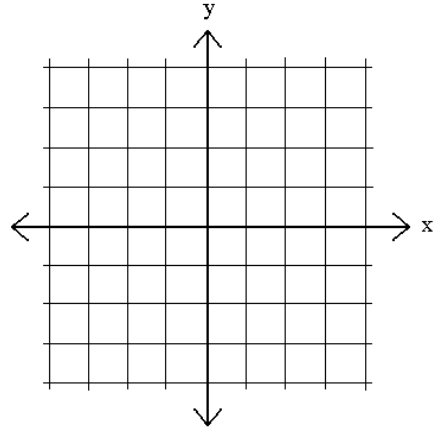
13. $y = x + 3$



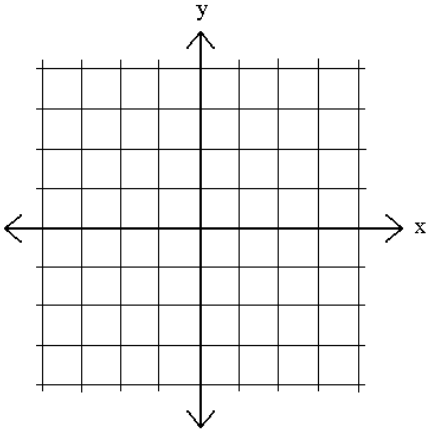
14. $y = 2x - 1$



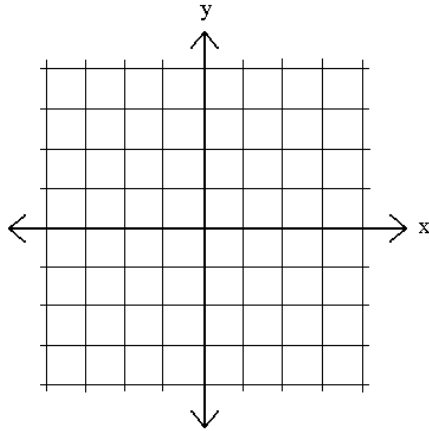
15. $y = 4x + 4$



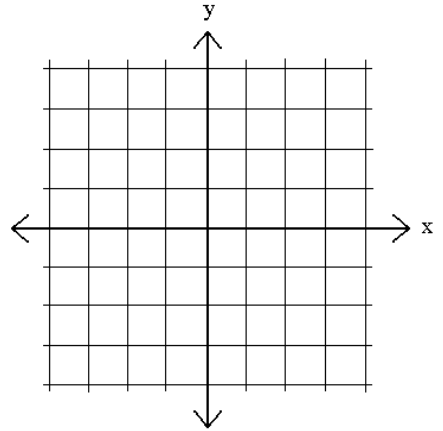
16. $4x + 5y = 15$



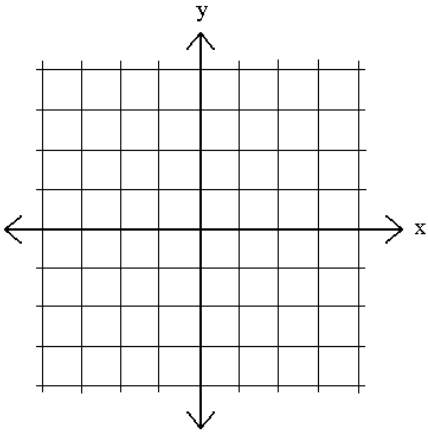
17. $4x - y - 3 = 0$



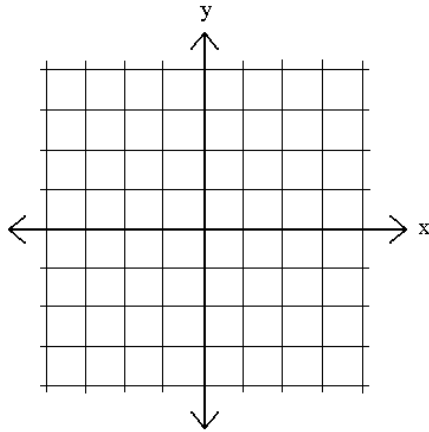
18. $5x + 15 + 5y = 10x$



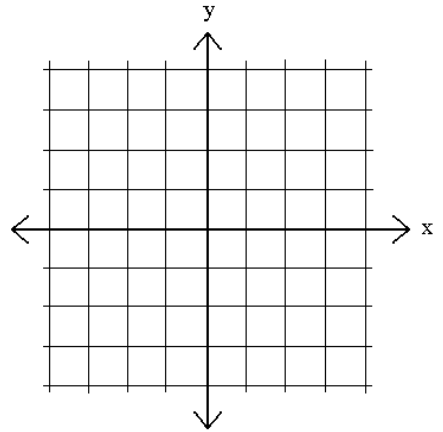
19. $2x + 2y - 4 = x + 5$



20. $-x + 2y = 6$



21. $2x - 3y - 6 = 0$



ANSWERS

1. $3x + y = -1$

$y = -3x - 1$

2. $10x - 5y = 50$

$y = 2x - 10$

3. $y - 4x = 9$

$y = 4x + 9$

4. $3x - 6y = 18$

$y = \frac{1}{2}x - 3$

Find the slope and y-intercept of the following equations:

5. $y = 3x - 7$

Slope: 3
y-int: -7

6. $y = 2x - 9$

Slope: 2
y-int: -9

7. $y = -2$

Slope: 0
y-int: -2

8. $y - 9x = 0$

Slope: 9
y-int: 0

9. $12x + 4y = 24$

Slope: -3
y-int: 6

10. $3x + 4y = 16$

Slope: -3/4
y-int: 4

11. $7y - 14x = 28$

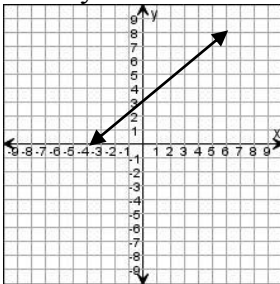
Slope: 2
y-int: 4

12. $-y = 2x + 3$

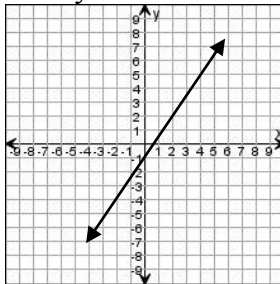
Slope: -2
y-int: -3

Graph the following equations:

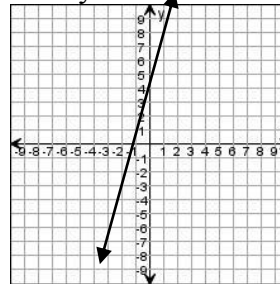
13. $y = x + 3$



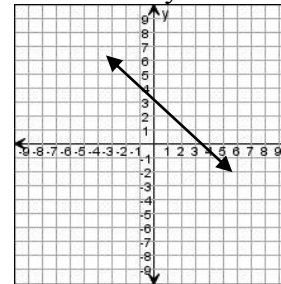
14. $y = 2x - 1$



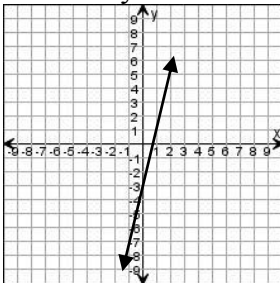
15. $y = 4x + 4$



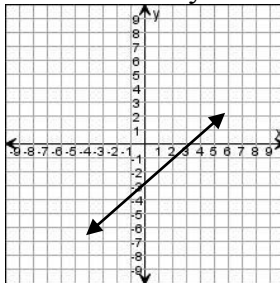
16. $4x + 5y = 15$



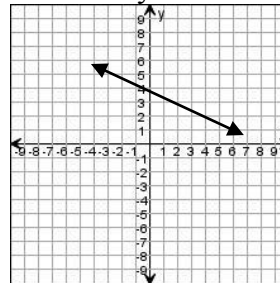
17. $4x - y - 3 = 0$



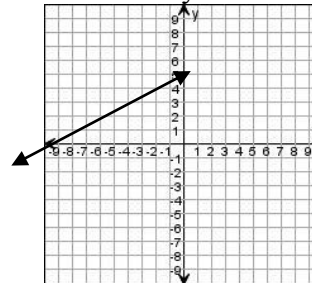
18. $5x + 15 + 5y = 10x$



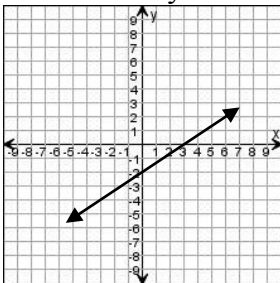
19. $2x + 2y - 4 = x + 5$



20. $-x + 2y = 6$



21. $2x - 3y - 6 = 0$



4. Writing Linear Equations

After doing all that slope-intercept stuff, now we can actually go backwards. That is, instead of taking an equation and graphing it, we can now take the graph and make an equation with it.

Example 1: Write the equation of the line whose slope is 3 and whose y-intercept is -4

Step 1: Write the slope intercept form: $y = mx + b$

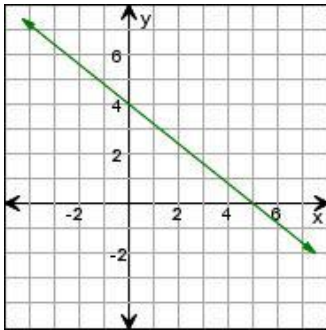
Step 2: $m =$ slope and $b =$ y-intercept

So substitute 3 for m , and -4 for b $y = (3)x + (-4)$

Step 3: Simplify, making the equation of the line: $y = 3x - 4$

Of course it's not always that easy...sometimes you have to find the slope.

Example 2: Write the equation of the line **Step 1:** Find the slope of the line: $m = \frac{\text{Rise}}{\text{Run}}$



The Rise is 4 and the Run is 5: $\frac{\text{Rise}}{\text{Run}} = \frac{4}{5}$

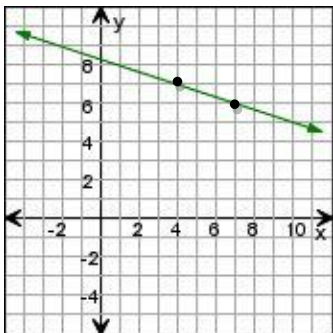
And since the line is going down from left to right, we know it's a negative slope. So the slope is $-\frac{4}{5}$

Step 2: Find the y-intercept: It crosses the y-axis at (0,4) so the y-intercept is 4

Step 3: Replace the values for m and b into the slope-intercept form: $y = -4/5x + 4$

And sometimes it's even harder....what if you don't know the y-intercept?

Example 3: Write the equation of the line



Step 1: Find the slope. rise = -1 run = 3 so slope is $-1/3$

Step 2: Plug in a point. So far you have $y = -1/3x + b$

In order to find b , you need to plug in an x and a y . So choose a point on the line!

There's a point at (4, 7) so $x = 4$, $y = 7$ Now you have: $7 = -1/3(4) + b$

Step 3: Solve for b . $7 = -1/3(4) + b$

$$7 = -4/3 + b$$

$$\underline{+4/3} \quad \underline{+4/3}$$

$$7 + 1 \frac{1}{3} = b$$

$$8 \frac{1}{3} = b$$

Now that you have your slope and your y-intercept, you can write the equation: $y = -1/3x + 8 \frac{1}{3}$

Write the equation of the line described below in slope-intercept form:

1. $m = 3$ $b = 2$

2. $m = 1$ $b = -1$

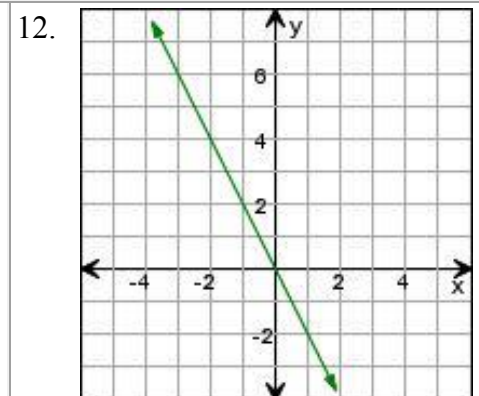
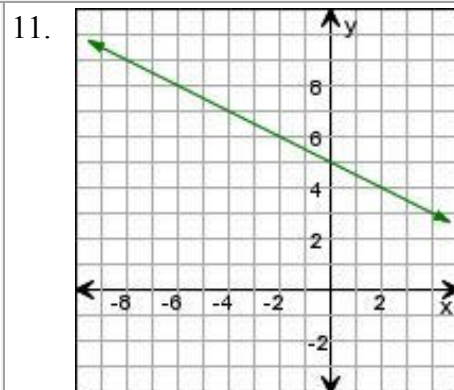
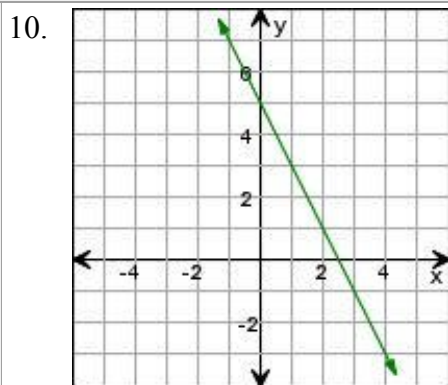
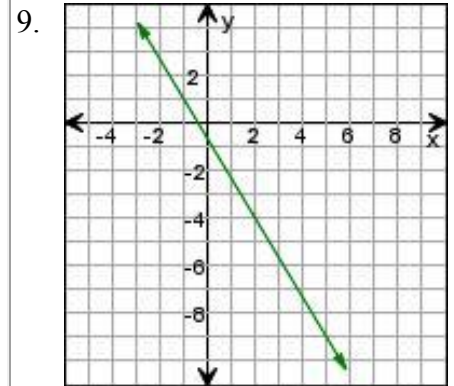
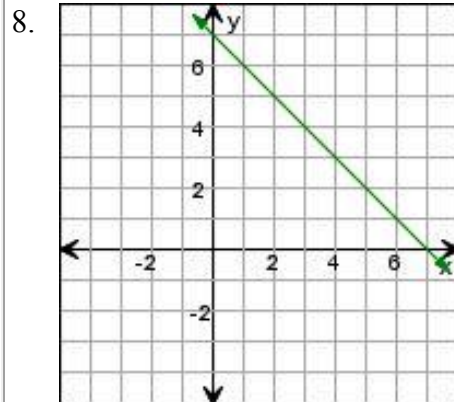
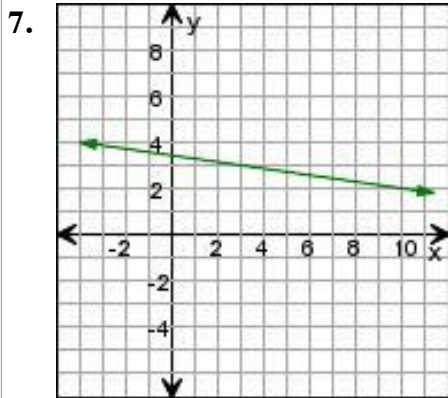
3. $m = -1$ $b = \frac{1}{2}$

4. $m = 0$ $b = 6$

5. $m = 10$ $b = 0$

6. $m = \frac{2}{5}$ $b = 7$

Write an equation for the following lines in slope-intercept form:



ANSWERS

Point slope form:

1. $y + 1 = 4(x + 1)$ 2. $y - 2 = -5(x + 6)$ 3. $y = 2(x + 10)$ 4. $y + 2 = 2(x + 8)$ 5. $y - 3 = -6(x + 4)$

Slope intercept form:

1. $y = 4x + 3$ 2. $y = -5x - 28$ 3. $y = 2x + 20$ 4. $y = 2x + 14$ 5. $y = -6x - 21$
6. $y = -x + 4$ 7. $y = -1/2x + 4$ 8. $y = -x - 3$ 9. $y = 2x - 20$ 10. $y = -1/3x + 8 \frac{1}{3}$
11. $y = -1/3x - 4 \frac{1}{3}$ 12. $y = -x - 7$ 13. $y = 4x - 15$ 14. $y = -2/7x + 6 \frac{5}{7}$

So now you've got two methods to write an equation!

The **slope-intercept** form is good if you're given the **slope** and the **y-intercept**

The **point-slope** form is good if you're given a **point** and the **slope**

But what if they don't even give you a graph, or the slope, or the y-intercept? What if they only gave you two points on a line, and you needed to figure out an equation? NO PROBLEM!!! Here's how:

Example: Write an equation of the line that contains the points, (3, -2) and (6, 0) Put your answer in slope-intercept form

Step 1: We need to find the slope: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{3 - 6} = \frac{-2}{-3} = \frac{2}{3} = m$

Now you have a slope and two points. Pick your method!

Point - slope way: $y - y_1 = m(x - x_1)$

Choose either one of the points and plug them into the equation. I'll use (6, 0)

$$y - 0 = 2/3(x - 6) \quad \text{or} \quad y = 2/3(x - 6)$$

Now to put it into slope-int form you just need to distribute: $y = 2/3x - 4$

Slope intercept way: $y = mx + b$

Choose either one of the points. I'll use (6, 0) again.

$$\begin{aligned} 0 &= 2/3(6) + b \\ 0 &= 4 + b \\ -4 &-4 \\ -4 &= b \end{aligned}$$

Same answer!!!

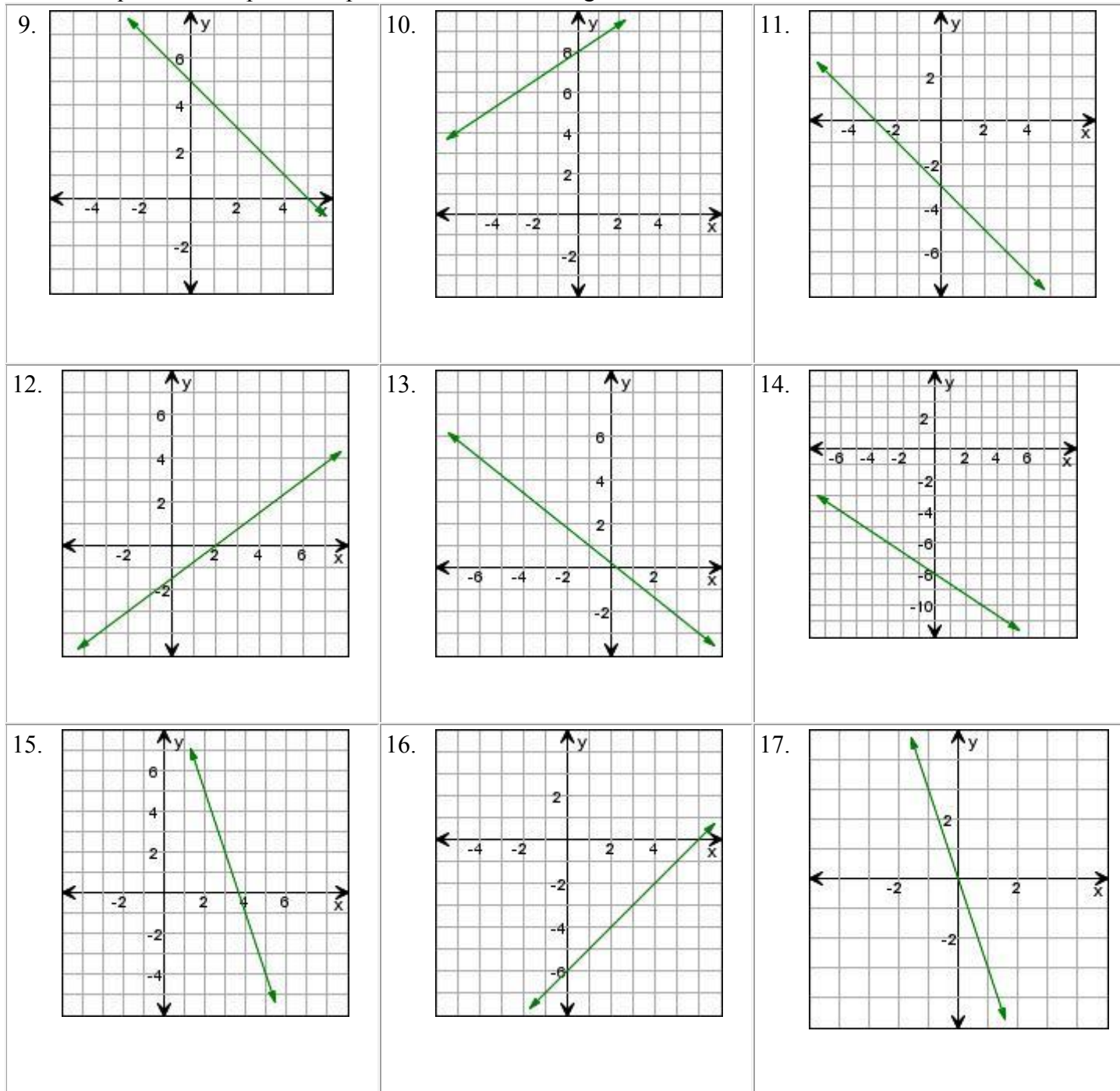
Now plug it in: $y = 2/3x - 4$

Now you try! Write the equation of the line that passes through the following two points in Slope-Int form:

1. (2, 3) and (0, 4) 2. (0, 0) and (-6, 5) 3. (0, 10) and (12, 4) 4. (0, 9) and (8, 7)

5. (11, -2) and (17, 6) 6. (-5, -7) and (2, -7) 7. (2, 0) and (-2, 6) 8. (2, -3) and (-3, 7)

Write an equation in slope-intercept form for the following lines ON A SEPARATE PIECE OF PAPER:



ANSWERS: 1. $y = -1/2x + 4$ 2. $y = -5/6x$ 3. $y = -1/2x + 10$ 4. $y = -1/4x + 9$ 5. $y = 4/3x - 50/3$
 6. $y = -7$ 7. $y = -3/2x + 3$ 8. $y = -2x + 1$ 9. $y = -x + 5$ 10. $y = 2/3x + 8$ 11. $y = -x - 3$
 12. $y = 3/4x - 3/2$ 13. $y = -4/5x + 1/5$ 14. $y = -2/3x - 8$ 15. $y = -3x + 11$ 16. $y = x - 6$ 17. $y = -3x$

One last form we need to tell you about is: **The Standard Form**

$$Ax + By = C$$

In the Standard Form, the variables are on the left side, and the constant is on the right:

An example would be: $2x + 5y = 7$

It's just another way that you may be asked to write an equation of a line. In Standard Form there are no fractions as coefficients.

Example: Put the following equation in standard form: $y = 9x - \frac{1}{2}$

Step 1: Clear any fractions by multiplying both sides by the least common denominator:
In this example, that would be 2:

$$2(y) = 2(9x - \frac{1}{2})$$

Step 2: Distribute

$$2y = 18x - 1$$

Step 3: Get all the variables on the SAME side

$$\begin{array}{r} 2y = 18x - 1 \\ -18x \quad -18x \end{array}$$

$$2y - 18x = -1$$

Step 2: Put the variables in alphabetical order:

$$-18x + 2y = -1 \quad \text{You're done!}$$

Now you try! Write the equations in standard form

1. $y = -5x + 2$

2. $y = 3x - 8$

3. $\frac{2}{9}y = -9 + 2x$

4. $y = \frac{2}{3}x$

5. $y = -\frac{3}{8}x$

6. $y = \frac{35}{x} + \frac{1}{2}$

7. $y = \frac{1}{2}x + \frac{3}{4}$

8. $3y = 2x$

9. $-\frac{3}{7}x = \frac{23}{y} - 5$

ANSWERS-----

1. $5x + y = 2$ 2. $-3x + y = -8$ 3. $-18x + 2y = -81$ 4. $-2x + 3y = 0$ 5. $3x + 8y = 0$ 6. $-6x + 10y = 5$
7. $-2x + 4y = 3$ 8. $-2x + 3y = 0$ 9. $-9x - 14y = -105$

BATTLESHIP MATH

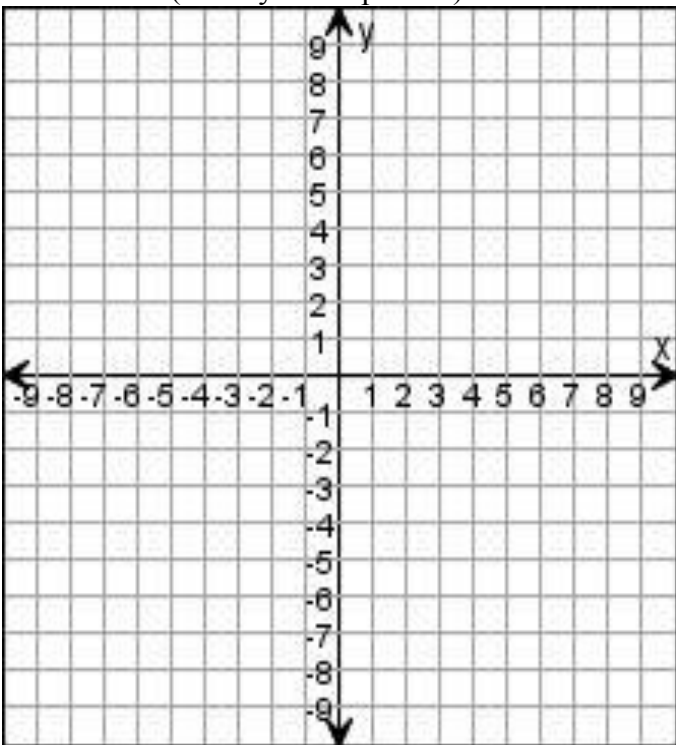
How to play: Pair up with a partner. Secretly mark off 5 total ships on your grid using a series of dots.

Your ships are: 1 Carrier – which is 5 dots long
 2 Destroyers – which are 4 dots long
 2 Submarines – which are 3 dots long

- You may place your ships either horizontally or vertically, but NOT diagonally
- Put a dot on each point to represent your ship, 5 dots for a carrier, 4 dots for a cruiser, 3 dots for a submarine
- Once you've placed your ships, you can flip a coin to decide who goes first.
- Players alternate one turn each, regardless of hit or miss
- The 1st player calls out a coordinate on the graph to represent a bomb being thrown
- If it happens to be the coordinate of one of the opponent's dots, the opponent should yell "hit"
- If the coordinate does not touch any of the ships, the opponent will say "miss"
- Keep track of your misses and hits on the graph on the right, the Opponent's Shipyard. Make a circle for a miss and an x for a hit, so you know where you've fired your bombs
- Keep track of your own ships on the graph on the left, called the Shipyard. Mark an X each time your opponent hits one of the dots on your boats.
- Once your opponent has successfully called out all of the points on a given ship, you must communicate that he sunk your battleship.
- The first person to sink all 5 of the opponents ships wins....or the person who has sunk the most ships by the time class ends wins.

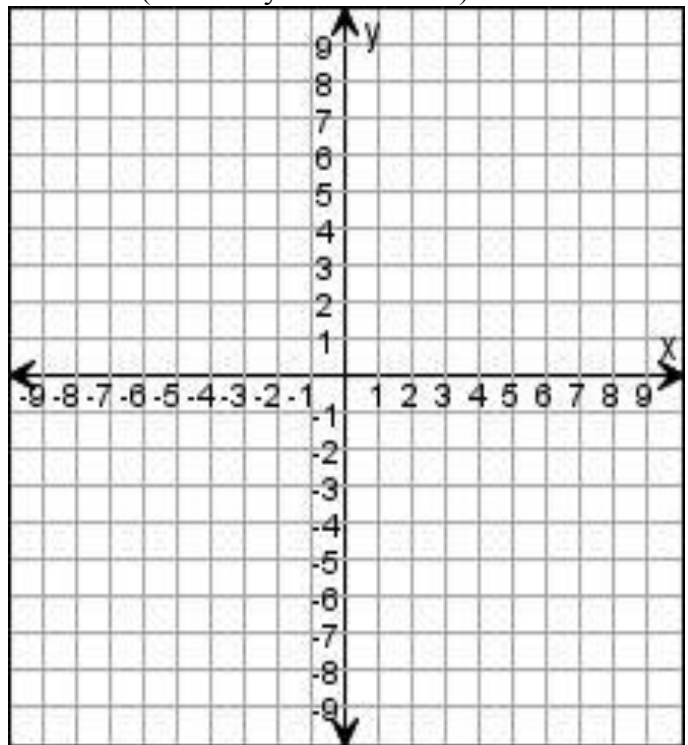
YOUR SHIPYARD

(Place your ships here)



THE OPPONENT'S SHIPYARD

(Monitor your shots here)



After you have completed your game, you need to write an equation for all 5 of your ship's positions. If you were to connect the dots on each one of your ships, making a line that continued infinitely both ways, what would be the equation of the line that each one makes?

Ships	Equation of line

What did you notice about the equations of each of these lines? Did it help more to use the Slope Intercept form or Point Slope Form?