8th grade
Orange Belt Training: Functions Discipline

Order of Mastery: MODELING
(8.F.4-5)
1. Linear models
2. Line of best fit
2. Dependent vs Independent variables
3. Algorithms

Welcome to the Blue Belt – Modeling! In case you were wondering it has nothing to do with clothes or taking pictures on a catwalk. I'm talking about modeling linear equations. It's the reason we needed to learn how to graph and write functions. This is what connects the seemingly pointless x's and y's with the adult world. You'll learn how to use graphs to predict stock growth on Wall Street. Do you ever hear those computer wizards talk about writing algorithms on TV shows and movies? An algorithm is simply a step by step procedure for solving a problem. Ever wonder how Google knows what you're looking for? It uses an incredibly sophisticated algorithm. Computer programmers write algorithms for everything from predicting the weather to finding your soulmate. That's what this belt is about. Recognizing relationships between variables.

Good Luck Grasshopper.

Standards Included:

CCSS.Math.Content.8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

CCSS.Math.Content.8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Modeling Linear Equations

A **LINEAR MODEL** is a linear equation that is used to model a real-life situation. A **RATE OF CHANGE** compares two things that are changing. Slope is often used to describe a real-life **RATE OF CHANGE.**

Example: From 1985 to 1997, the number of movie theaters in the United States increased by about 750 per year. In 1993, there were about 26,000 theaters. Write a linear model to represent the increase of theaters per year.

In this case, our x-axis represents the year, and the y-axis represents the amount of theaters. That means we have a point: (1993, 26,000). For the slope, the question said the theaters increased by about 750 theaters per year. That's our RATE OF CHANGE, which is our slope. So slope = 750. Now we can write an equation, using any way we can:

The point-slope formula way: OR The slope-intercept way:

$y - y_1 = m(x - x_1)$ $y = mx + b$

$y - 26{,}000 = 750(x - 1993)$ $26{,}000 = 750(1993) + b$

$y - 26{,}000 = 750x - 1494750$ $26{,}000 = 1494750 + b$

$+26{,}000 \quad +26{,}000$ $-1494750 \quad -1494750$

$y = 750x - 1468750$ $-1468750 = b$

$y = 750x - 1468750$

Example: How many theaters were there in the year 2005?

Now that we have an equation, 2005 is a year, which is represented by x. So x = 2005. Just plug it into our equation to solve for the number of theaters (y).

$y = 750(2005) - 1468750$ $y = 1503750 - 1468750$

y = 35,000 theaters in the year 2005

Now you try!

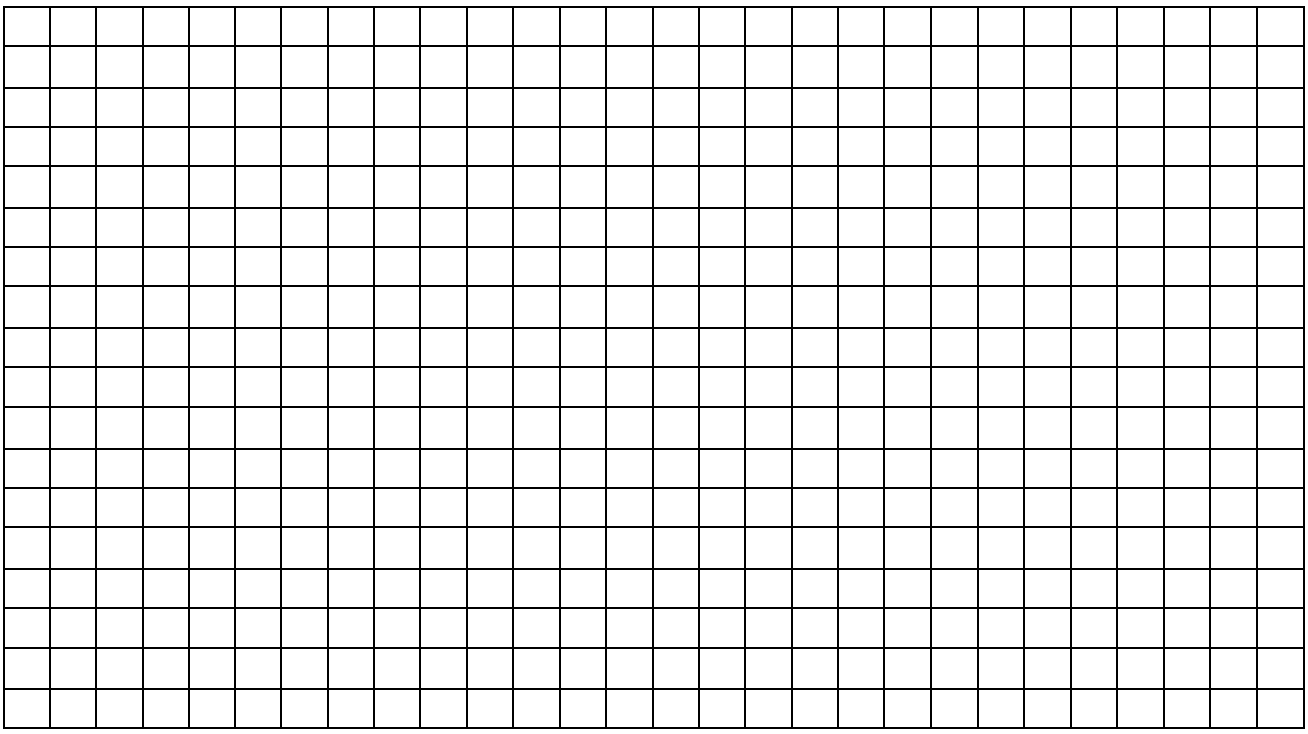
1. From 1986 to 1999 Disney had an increase in sales of their Mickey Mouse dolls by 900 per year. In 1996, the company sold 17,000 Mickey dolls.

   a. Write a linear model for the number of stuffed animals sold (y).

   b. Predict how many Mickey dolls will be sold in the year 2006.

   c. Graph the linear model on the next page

Graph it:



2. **MOUNTAIN CLIMBING:** A mountain climber is scaling a 400 foot cliff. The climber starts at the bottom and climbs at a constant rate of 124 feet per hour.

a. Should the number of feet climbed represent the x-axis or y-axis? Explain why.

b. What is the slope in the linear model for this situation?

c. The y-intercept represents the height at which the climber begins scaling the cliff. What is the y- intercept in this model?

d. Use the slope and the y-intercept to write the linear model for the distance that the climber climbs in terms of time in hours.

e. After 3 hours, has the climber reached the top of the cliff?

f. Use the equation you found to determine the time the climber will reach the top of the cliff.

3. CANOE RENTAL: Renting a canoe on the Gallatin River in Montana costs $10, plus $28 per day to use.

a. If the total cost is represented by y, and the number of days rented is represented by x, what number corresponds to the slope?

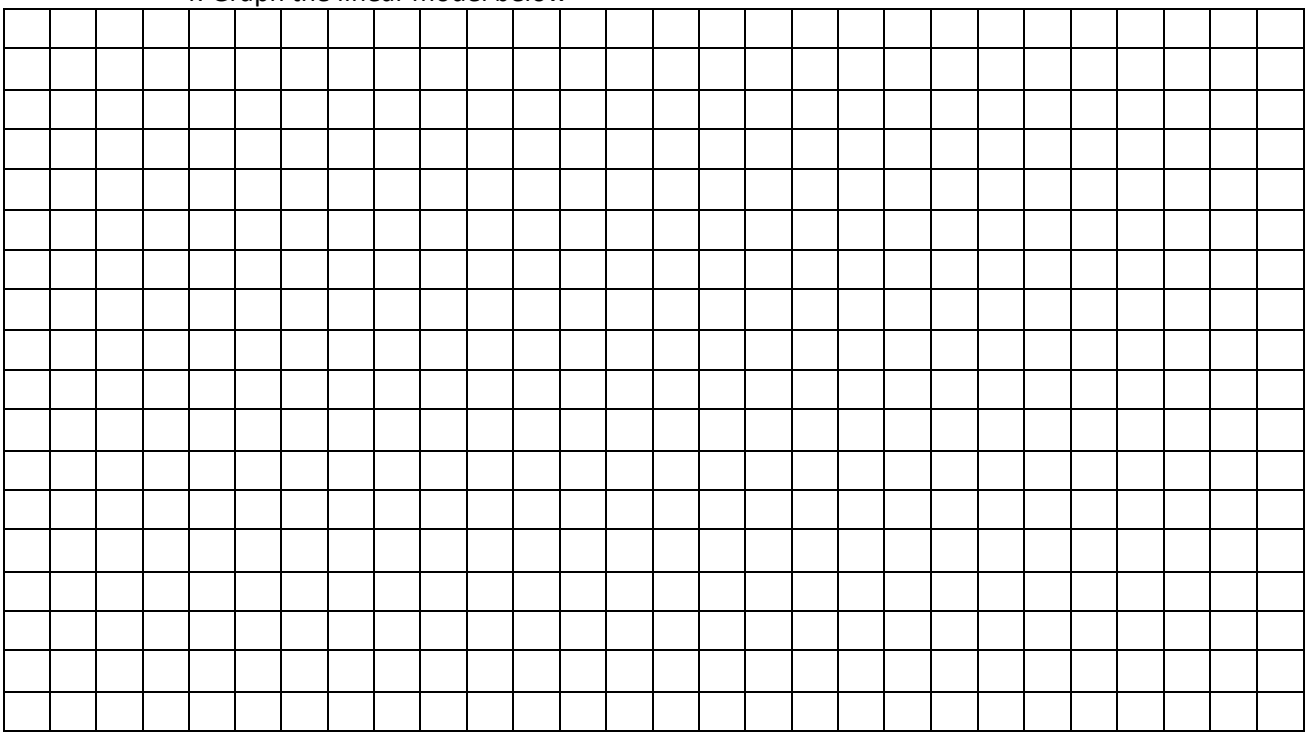
b. What number corresponds to the y-intercept?

c. Now that you know those two things, write the linear model of this problem.

d. What would be the cost of renting a canoe for 5 days?

e. If you had $66 to spend, for how many days could you rent a canoe?

f. Graph the linear model below



4. CAR COSTS: From 1994 through 1997, the cost of owning and operating a car per mile, which includes maintenance and repair, increased by about 2.2 cents per year. In 1995, it cost about 48.9 cents per mile to own and operate a car.

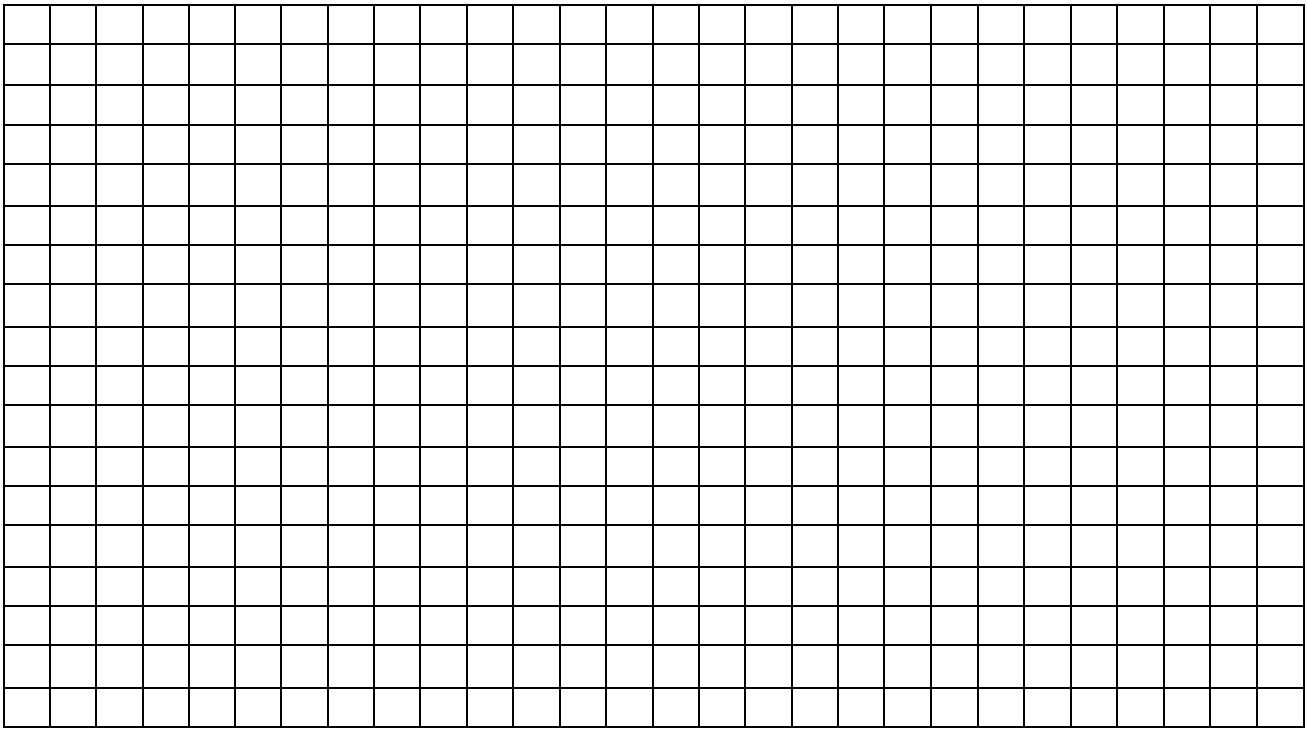
a. What's the slope of the linear equation that models this situation?

b. Name one point on the line.

c. Use the slope and that point to make an equation for the linear model. Let C represent the cost of owning and operating an automobile in terms of time (t).

d. Predict the cost of owning and operating a car (per mile) in 2006.

e. Graph the linear model below



ANSWERS

1) a. y = 900x - 1779400 b. y = 26,000
2) a. v — axis This is a distance/time graph, so in order to calculate the rate, the feet should be y, and the time should be x.
b. 124 c. 0 d. v = 124x e. no f. 3.2 hours
3) a. 28 b. 10 c. y = 28x + 10 d. $150 e. 2 days
4) a. 2.2 b. (1995, 48.9) c. C — 48.9 = 2.2(t — 1) or C = 2.2t - 4340.1 d. 73.1 cents per mile

Why do I need to know what an x and y intercept is?

Because I say so! Just kidding. In financial models, the intercepts can help you decide how much money to charge each person for things like plays, movies, amusement parks, etc. They don't just randomly come up with a price for that Disneyland ticket. It's all about the money. And this is where you come in.

The assignment: You're an intern at the San Diego Zoo. Your boss says he'll hire you for good if you can find a way to raise $1500 at Friday's Nighttime at the Zoo exhibit. He expects there will be 200 adults and 100 students coming to the exhibit. How much should you charge for an adult ticket and how much should you charge for a student ticket? There's more than one answer to this problem. You'll have to make a presentation to him to show the different prices you could use for each ticket and still make $1500. This means you need to make a graph.. ..and figure out some other stuff.

The process: Your table group will need to answer the following questions regarding this financial issue.

a. Write a linear equation to model the situation. (use x and y as your two unknown variables)

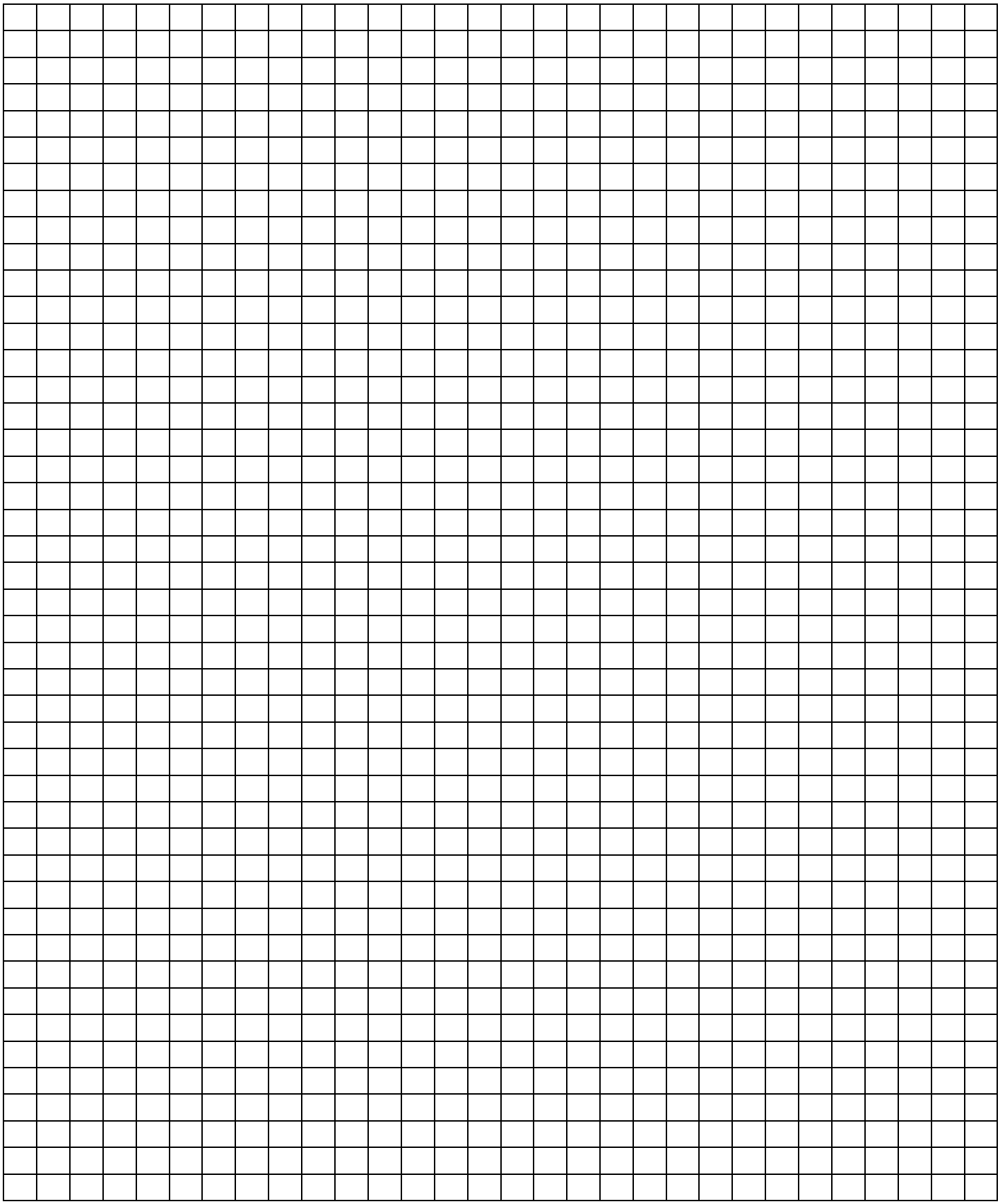
b. What is the x-intercept? What does it represent in this situation?

c. What is the y-intercept? What does it represent in this situation?

d. If students cannot afford to pay more than $3 for a ticket, what can you say about the price of an adult ticket?

e. What does your group think is the best amount to charge for an adult ticket and a student ticket? Explain why?

f. Graph your findings below. When you've completed all these questions, you will present your findings at the end of class to your boss, which happens to be me.



THE DANCE

You just joined ASB at school. You need to raise $750 to buy a new basketball hoop for the after school program. So you decide to have a dance, but you don't know how much to charge for adult tickets and how much to charge for student tickets. You expect about 25 adults will come, and 225 students will come.

1. Write a linear equation to model this scenario: (use x and y for variables)

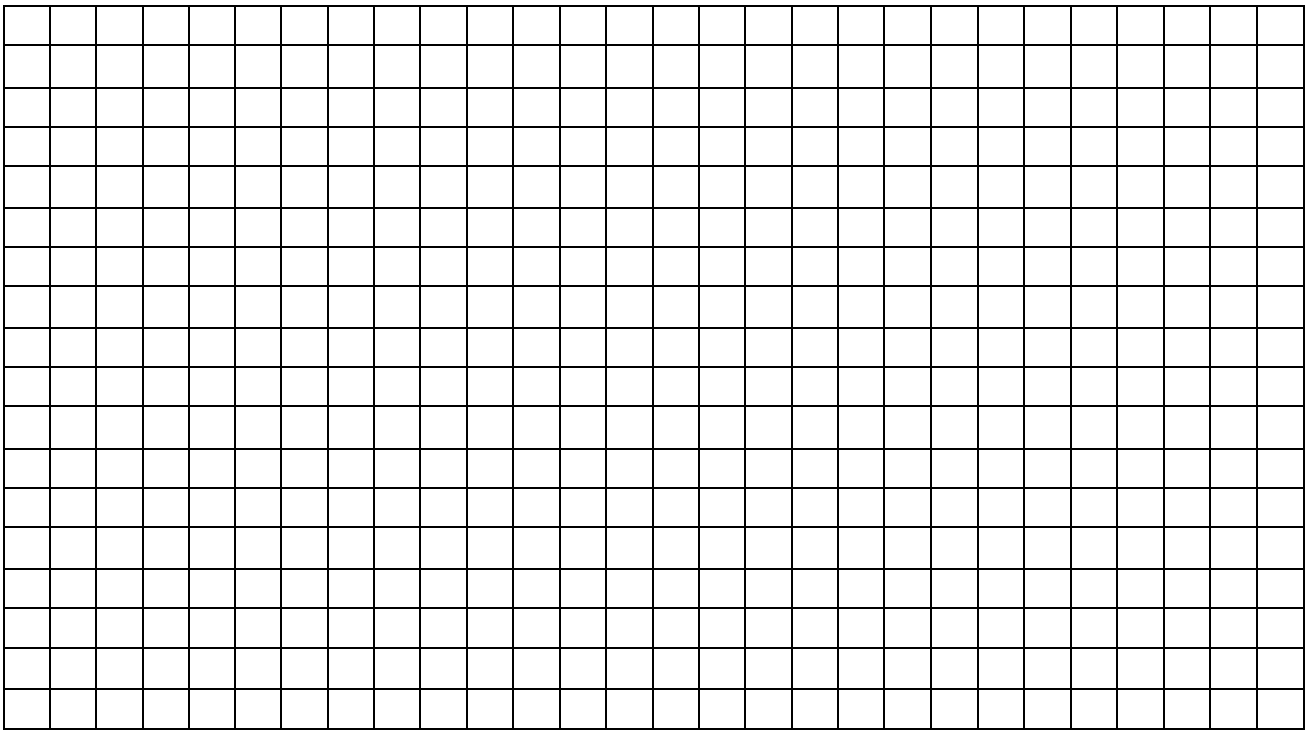
2. What's the x intercept?

3. What's the y intercept?

4. What's the rate of change?

5. The students held a protest because they had to pay for tickets. They said they won't pay more than $1.50 per ticket. If that's the case, how much will you have to charge the adults?

6. Graph the linear model:



THE HALO DILEMMA

You tried to buy your favorite video game online, but when you filled out the form, you made a mistake. You accidentally bought 35 versions of HALO, and 15 versions of HALO 2. And they charged you $1625. So you decide to sell them. How much should you charge for each different version in order to make your $1625? Since HALO 2 is way more expensive than HALO 1, you need to sell them at different prices.

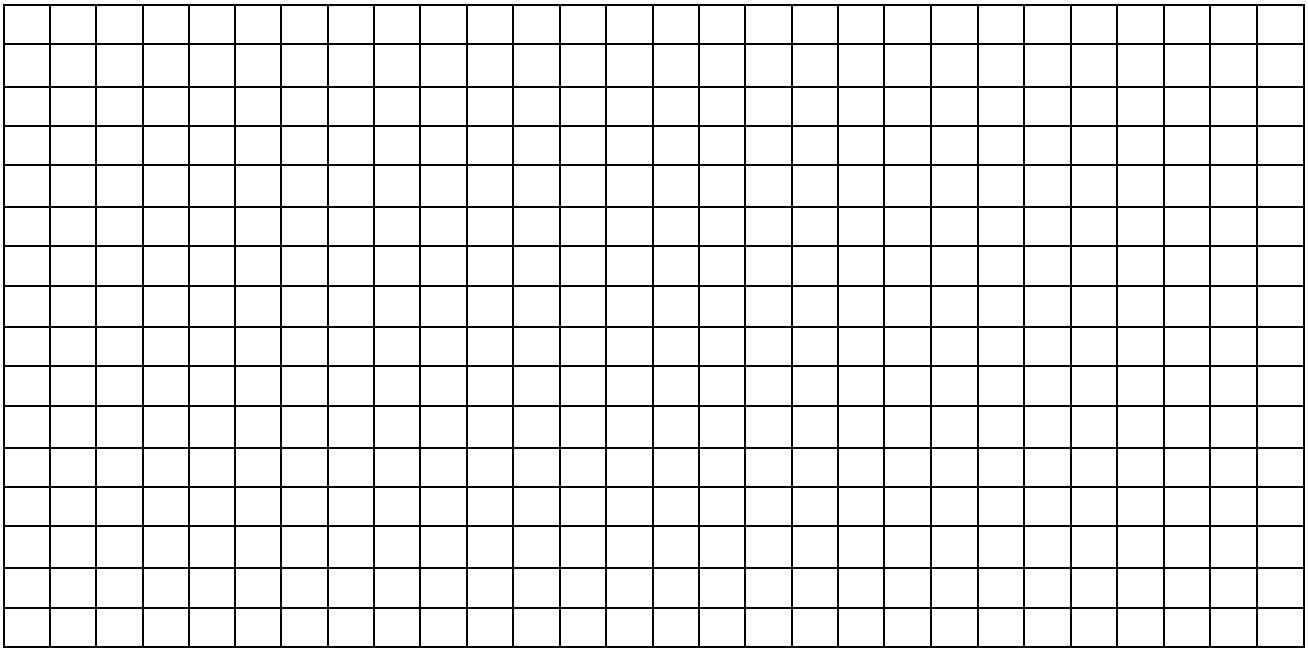
1. Write a linear equation to model this scenario: (use x and y for variables)

2. What's the x intercept?

3. What's the y intercept?

4. What's the rate of change?

5. Graph the linear model:



ANSWERS

ZOO

a. x = parent price y = student price 200x + 100y = 1500 or 2x + y = 15 b. graph c. The x - intercept is at (7.5, 0) It represents the price the parents would pay if the students got in free $7.50 d. The y-intercept is at (0, 15) It represents the price the students would pay if the parents got in free e. The adults would have to pay at least $6 a ticket in order to earn $1500 total. f. Answer varies

DANCE 25x + 225y = 750 or x + 9y = 30 x = price of adult tix y = price of student tix. You'll have to charge the adults at least $16.50 in order to make the 750 dollars

HALO x = halo 1 y = halo 2 35x + 15y = 1625 or 7x + 3y = 325 (46, 1) (7, 92)

COOL SHOES.COM

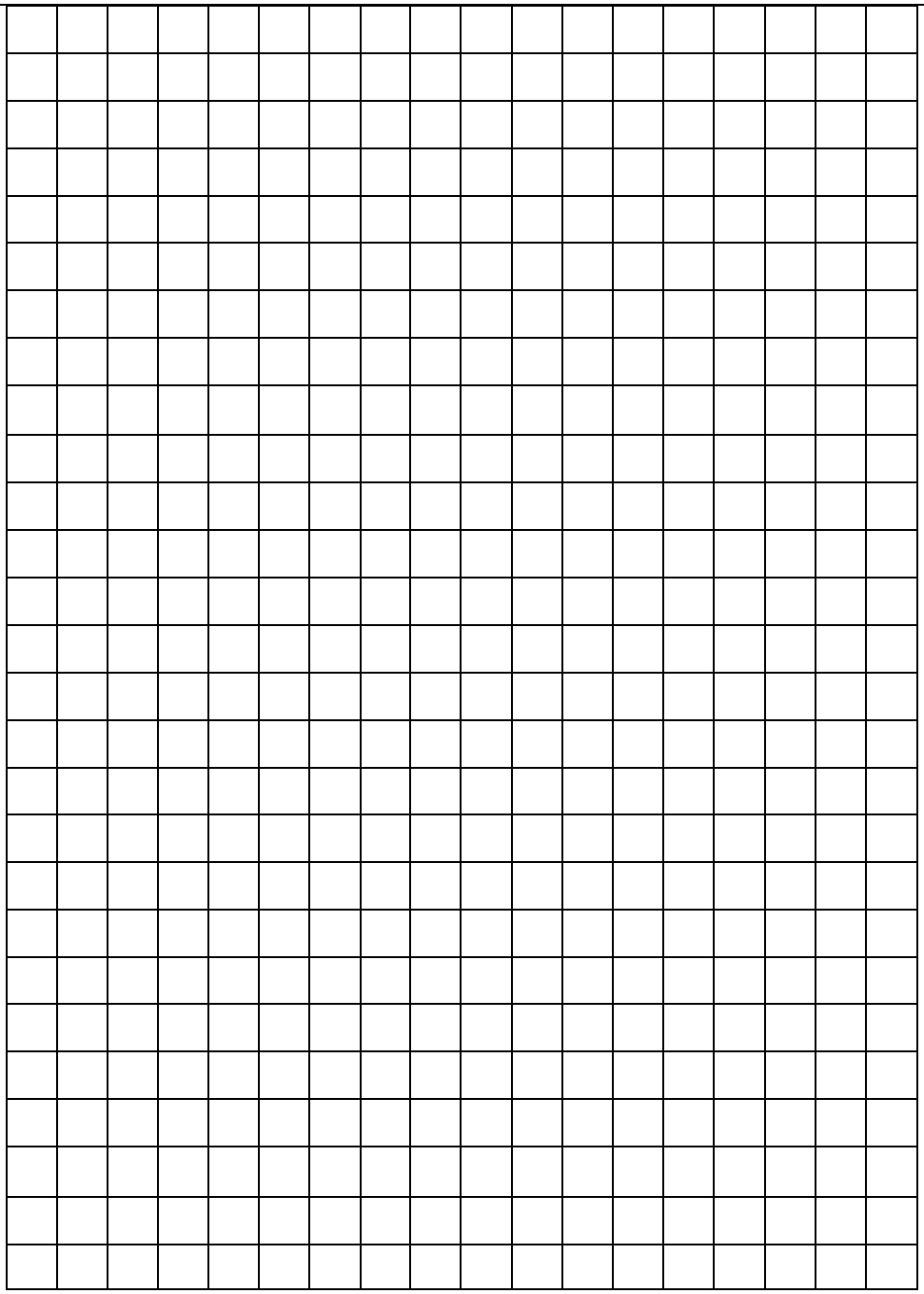
MPJ's Ultimate Math Lessons

Adapted from CPM Educational Program's project "Bee Bopper Shoe Store"

You own and operate CoolShoes.com, an online shoe store. Many people want to order shoes for friends and relatives, but do not know their shoe size. Since it's easier to estimate a person's height than shoe size, you want the customer to be able to enter a person's height and calculate the appropriate shoe size (approximate). You must have either a graph or equation in order to do this. So, your task here is to create both, using sample data from your class.

BOYS

Height (inch)	Shoe Size



GIRLS

Height (inch)	Shoe Size

1. Fill in the charts with data from your class. Record each person's height and corresponding shoe size.

2. Plot the data points from the charts. Use (+) to plot each of the boys points, and a normal dot to plot the girls so you can tell them apart.

3. Do you notice any relationship between people's height and their shoe size? What kind of correlation is it?

4. Draw an approximate line of best fit for each set of data (one for the boys, one for the girls)

5. For each line, calculate the rate of change (slope)

BOYS: There exists a change of _____sizes for every _____inches of height.
That means there's a change of _____sizes per every one inch.

GIRLS: There exists a change of _____sizes for every _____inches of height.
That means there's a change of _____sizes per every one inch.

6. a) Calculate the y-intercept of each line. BOYS_____ GIRLS _____

b) What do these intercepts imply? Do they match your graph?

7. Write the equation of each line.

BOYS _____ GIRLS_____

8. For each set of data, find a height that does NOT appear in the chart. For instance, if no girl in the class is exactly 68 inches tall, then choose 68 inches for the girls. Use your equation and your chosen value for height to find the corresponding shoe size at that height. Do your solutions match your graph?

BOYS Height_____ GIRLS Height_____
Shoe Size _____ Shoe Size_____

9. For each set of data, find a shoe size that does NOT appear in the chart. For instance, if no boy in the class has a shoe size of 13.5, then choose 13.5 for the boys. Use your equation and your chosen value for shoe size to find the corresponding height. Do your solutions match the graph?

BOYS Height_____ GIRLS Height_____
Shoe Size _____ Shoe Size_____

THE WAVE

Equipment: 1 stopwatch, large group of people, ruler

Start with a group of 5 students. The timer says go, and students make a wave (like the one at a stadium during a sporting event) To do this, the first student stands up and sits down while waving their hands. Once the first student starts to sit down, the second students repeats the process, and so on. Once the last person sits down, stop the clock. Repeat this experiment with 8, 10, 15 and 18 students. If you can get more to participate, great! Collect your data below and then graph it!

The independent variable (x) in this experiment is _____ Units _____

The dependent variable (y) in this experiment is_____ Units _____

DATA

X	Y

After plotting your data, draw a straight line through two of your points to approximate the line of best fit. Choose the line that best fits your data. Circle the points on the graph and copy their coordinates below:

Your points: (_____, _____) and (_____, _____)

Use these points to find the equation of your line. Show all your work.

Find the slope of line: **Find the y-intercept of the line:**

Write the equation of the line. **y = _____x + _____**

Rewrite the equation of the line, using the names of the variables instead of x and y

_____ = _____ + _____

Interpret the data:

1. How long would it take 40 students to make a wave? _____

2. How many students are needed for a 25 second wave?_____

3. Was your answer to Question 2 a whole number? _____ Does a non-whole number make sense for this answer?_____

4. How many students must get up and sit down for a 3 minute wave? _____

5. With a group of 33 students, how long would it take to make a complete wave? _____

6. How would your graph be different if every student stood up and turned around twice before sitting down?

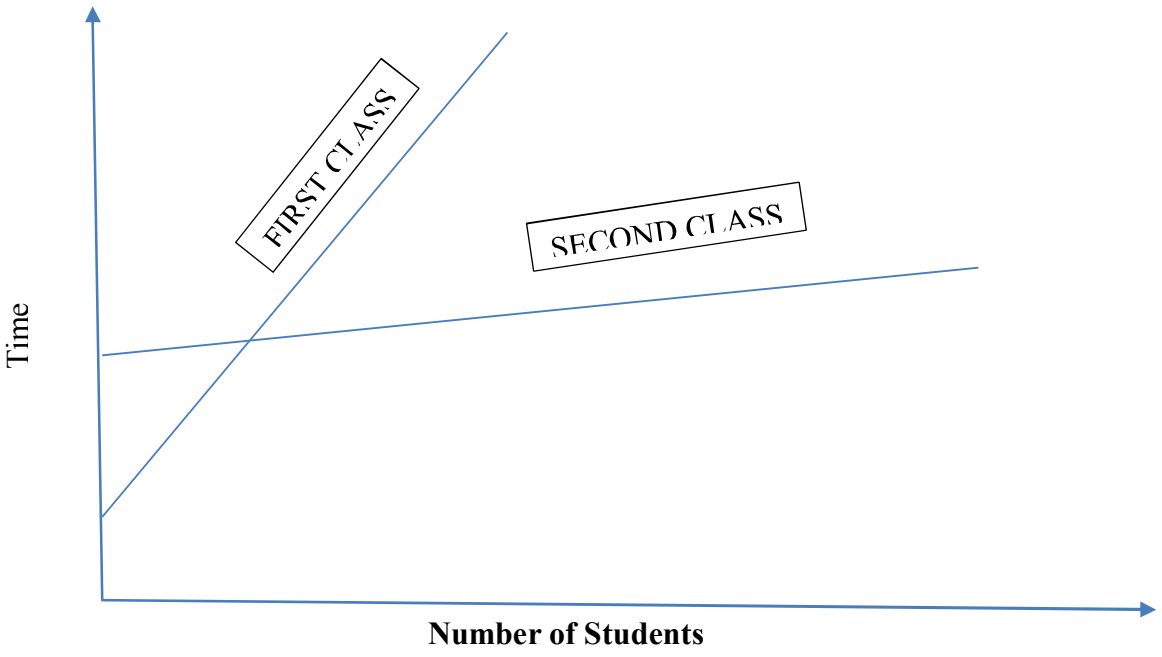
7. How could we change the experiment so that the line would have less slope?

8. How could we change the experiment so that the line would be steeper?

We did this experiment last year with both of my classes. The graphs of both classes are shown below.

9. Give a possible explanation of why the slopes are different:

10. Give a possible explanation of why the y-intercepts are different:



SHADOWS

Equipment: Tape measure, several objects of different sizes (books, folders, chairs, etc)

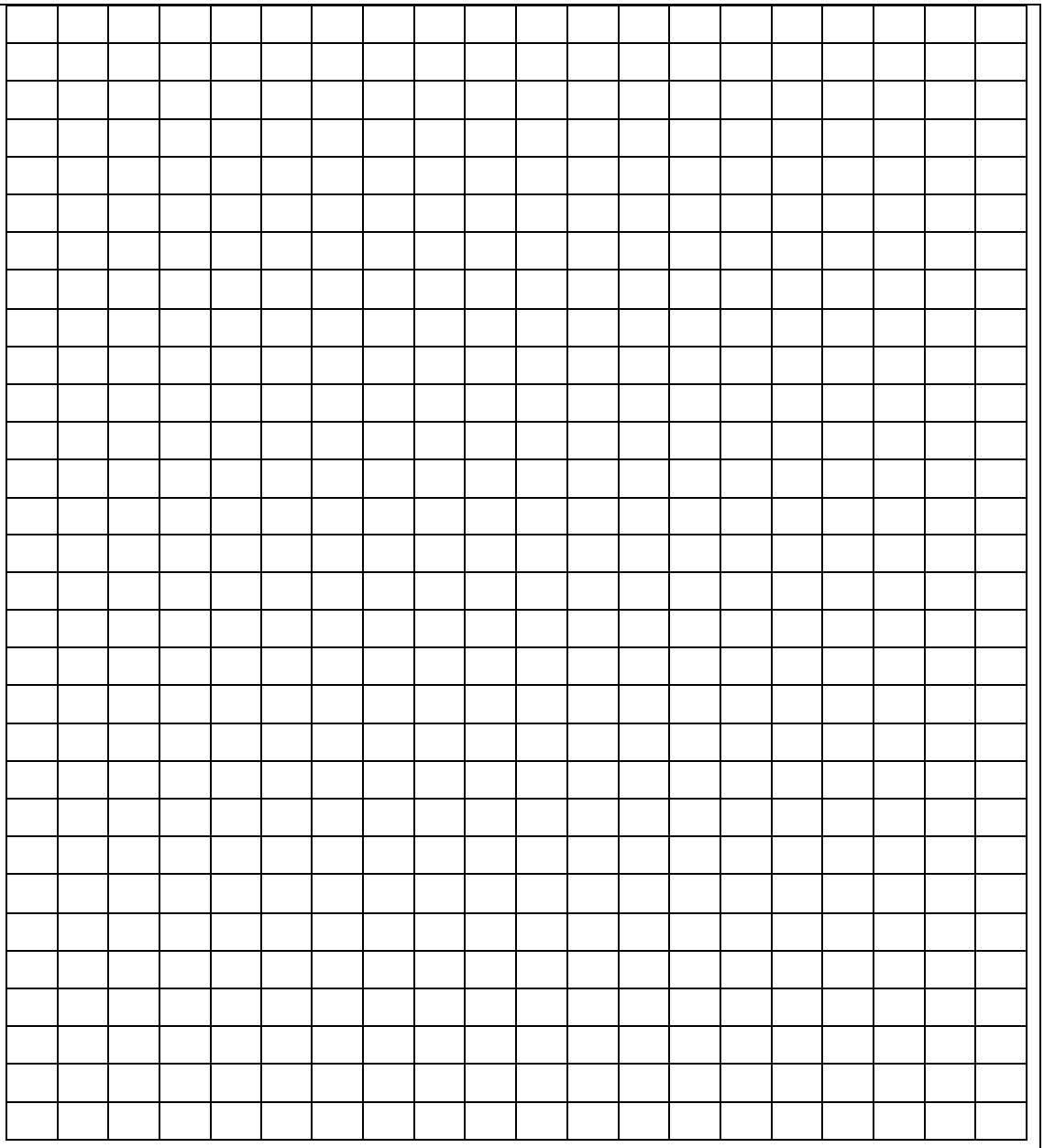
If there's no sunlight outside, it's really hard to do this experiment. You need to be able to see the shadow. So it's best not to do this experiment at noon. Use objects over 10 inches in height. You'll need to measure the height of each object along with the length of the shadow each object produces. Then plot your data below.

The independent variable (x) in this experiment is _____ Units _____

The dependent variable (y) in this experiment is_____ Units _____

DATA

X	Y



After plotting your data, draw a straight line through two of your points to approximate the line of best fit. Choose the line that best fits your data. Circle the points on the graph and copy their coordinates below:

Your points: (_____, _____) and (_____, _____)

Use these points to find the equation of your line. Show all your work.

Find the slope of line: **Find the y-intercept of the line:**

Write the equation of the line. **y = _____x + _____**

Rewrite the equation of the line, using the names of the variables instead of x and y

_____ = _____ + _____

Interpret

1. If the object you used were 91 in high, what would be the length of the shadow?_____

2. Find the height of an object that would make a shadow 43 inches in length_____

3. Find the height of an object that would make a shadow 61 inches in length_____

4. Explain how your line would be different if you did the experiment at noon:

5. At what time of day do you think the slope of your line would be the steepest? _____

At what time would it be the flattest?_____

6. Your classmate Juanita makes her measurements in centimeters instead of inches. If she does her experiment at the same time you do yours, will her line have the same slope as yours?_____
Will it be parallel or will it intersect?_____

7. Find a large building or flagpole at your school and measure the length of it's shadow. Then use your equation to determine the height. Ask the facilities whether you were correct.

BOUNCING BALL

Equipment: Tape measure, different types of balls (golf ball, tennis ball, racquetball, etc)

You'll need to be in a group of two for this one. The first student drops a ball from a given height (the drop height); the second student measures the distance from the top of the ball to the ground after it bounces (bounce height). For each drop height, you should drop the ball three times to get the average bounce height. That represents the most accurate result.

The independent variable (x) in this experiment is _____ Units _____

The dependent variable (y) in this experiment is_____ Units _____

DATA

X	Y

After plotting your data, draw a straight line through two of your points to approximate the line of best fit. Choose the line that best fits your data. Circle the points on the graph and copy their coordinates below:

Your points: (_____, _____) and (_____, _____)

Use these points to find the equation of your line. Show all your work.

Find the slope of line: **Find the y-intercept of the line:**

Write the equation of the line. **y = _____x + _____**

Rewrite the equation of the line, using the names of the variables instead of x and y

_____ = _____ + _____

Interpret

1. If you were to drop the ball from a height of 210 inches, how high would it bounce? _____

2. If your ball bounces 123 inches, what was the drop height?_____

3. If the height of your ball's bounce is 72 inches, what was the drop height? _____

4. How would your graph be different if you'd used a "super ball"

5. Julio's slope was 13/19 and Chris' slope was 14/29. Who used the tennis ball and who used the golf ball? How do you know?

6. If you were to drop the ball from a height of 210 inches, how high would the SECOND bounce be?

CIRCLES

Equipment: Tape measure, string, several circular objects of different sizes (jar lids, cans, etc), scissors

You'll need to find the circumference and diameter of several different circles. To find the circumference of each item, you merely need to wrap the string around it and then measure the length of the string. In order to find the diameter (y), trace around each object, cut out the circle on paper, then fold the circle exactly in half. The folded line is the diameter.

The independent variable (x) in this experiment is _____ Units _____

The dependent variable (y) in this experiment is_____ Units _____

DATA

X	Y

After plotting your data, draw a straight line through two of your points to approximate the line of best fit. Choose the line that best fits your data. Circle the points on the graph and copy their coordinates below:

Your points: (_____, _____) and (_____, _____)

Use these points to find the equation of your line. Show all your work.

Find the slope of line: **Find the y-intercept of the line:**

Write the equation of the line. **y = _____x + _____**

Rewrite the equation of the line, using the names of the variables instead of x and y

_____ = _____ + _____

Interpret

1. What is the circumference of a circle with a diameter of 14 inches? _____

2. What is the diameter of a circle with a circumference of 150 inches?_____

3. What is the diameter of a circle with a circumference of 42 inches? _____

4. According to your equation, what is the diameter of a circle with a circumference of 160 inches

5. Measure the circumference of a tree. What is it's diameter? _____

6. Otis measured large objects and used feet instead of inches for both variables. What effect would this have on the resulting equation and graph?

7. What is pi?

ALGORITHMS

Not sure what an algorithm is? Well here's the technical definition: a set of steps that are followed in order to solve a mathematical problem or to complete a computer process.

Why is it important? For one, it's how we use computers to solve incredibly difficult problems. Google's algorithm for its search engine is one of the most powerful algorithms ever. While we can't really write one like that in just a day or so, what we can do is write a simple algorithm to predict future stock growth.

In order to do that, we need to do a little research. How has that stock done over a longer period of time?

Look up a stock, any stock you want, on the Yahoo Finance Website. You're going to track this stock over the past 5 years to see where it's going. After you type in the symbol of your stock, look at the graph on the right. Select 5yr on the bottom so you can see how the stock has done over the course of 5 years. You'll then write a line of best fit like the one you've been doing in all the previous experiments. This will help you create an equation that will help you predict where it will be in the future.

But before you try creating an algorithm on your own, here a couple examples for you to practice on. I've provided a template for you to record the appropriate information to find the equation, just like we've been doing in the experiments:
After plotting your data, draw a straight line through two of your points to approximate the line of best fit. Choose the line that best fits your data. Circle the points on the graph and copy their coordinates below:

Your points: (_____, _____) and (_____, _____)

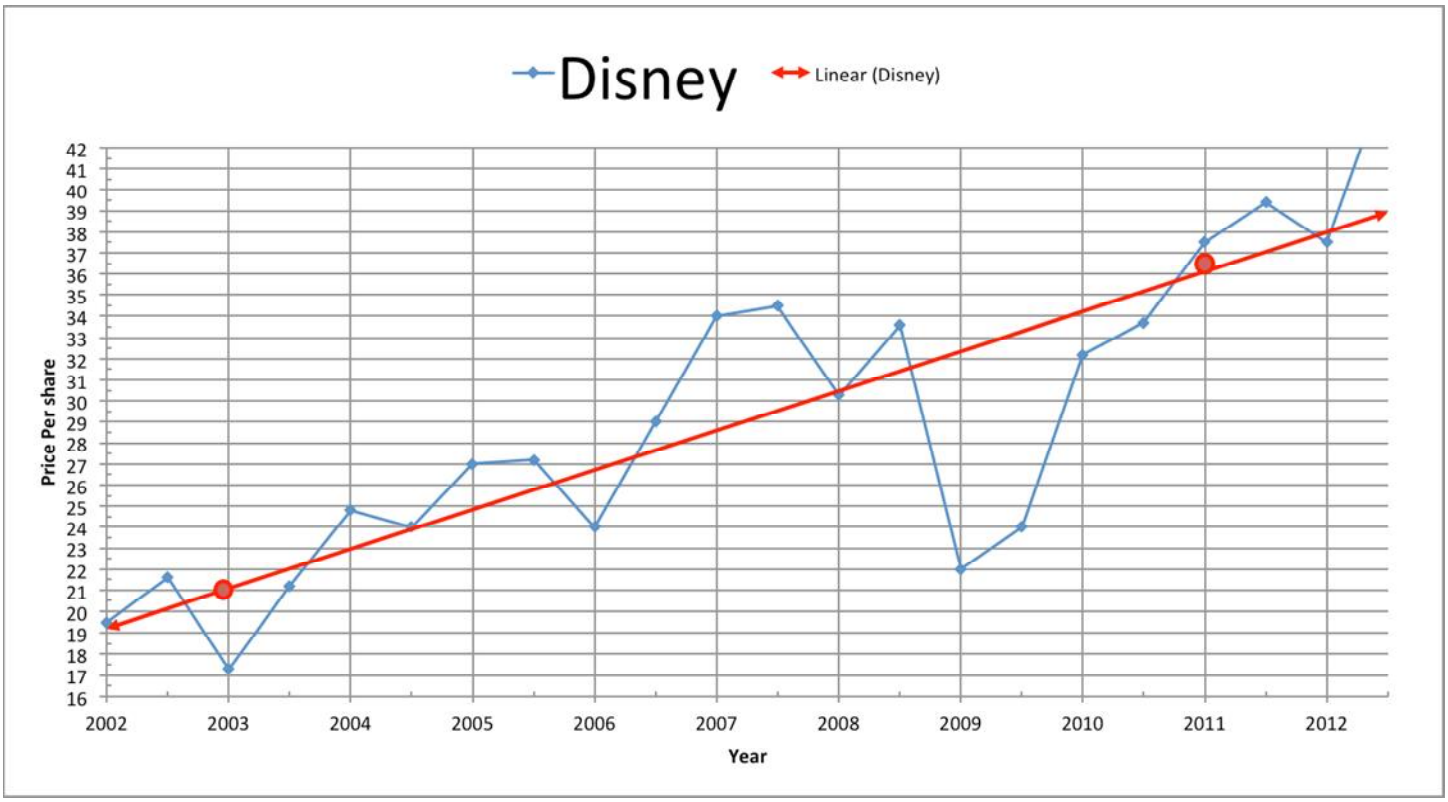
Use these points to find the equation of your line. Show all your work.

Find the slope of line: **Find the y-intercept of the line:**

Write the equation of the line. **y = _____x + _____**

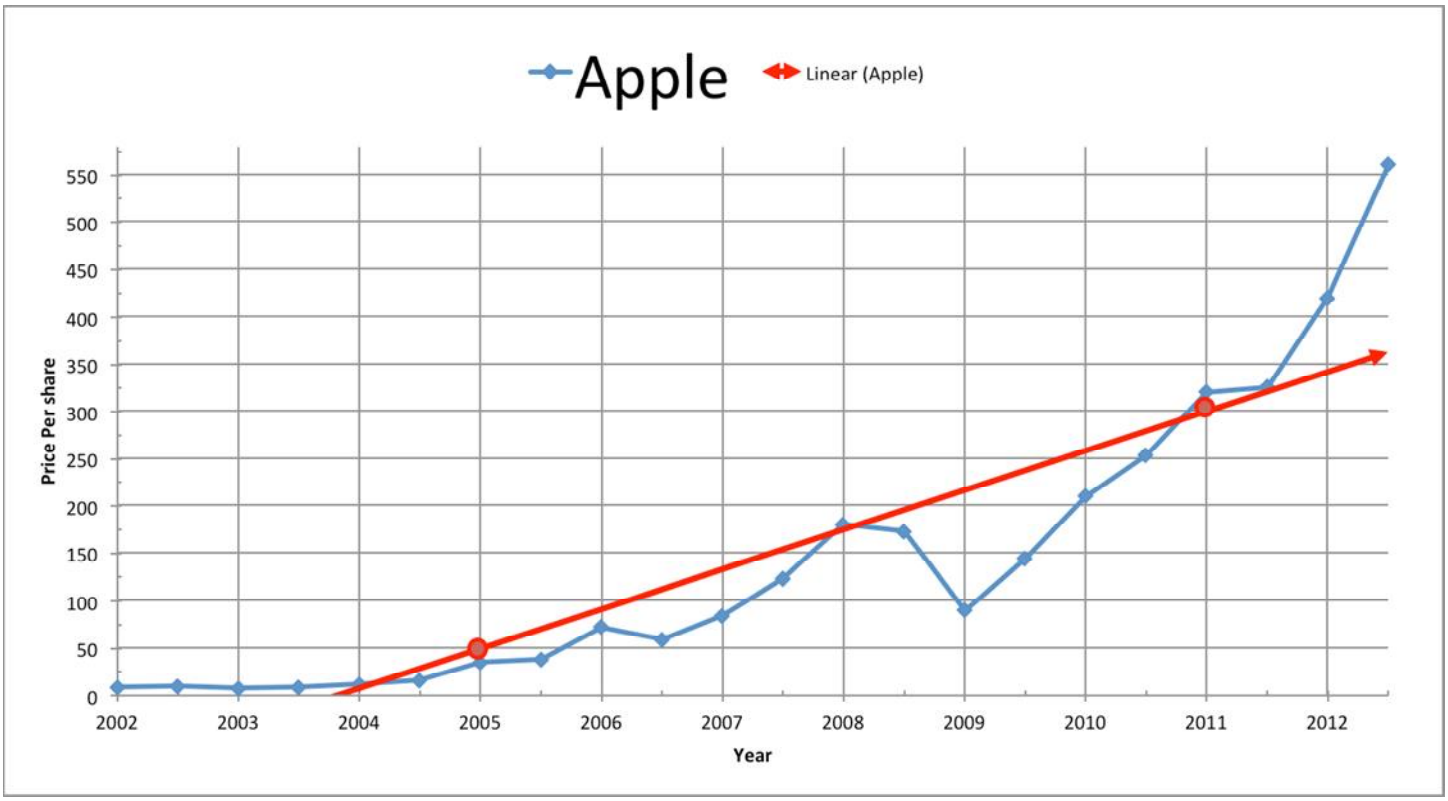
Rewrite the equation of the line, using the names of the variables instead of x and y

_____ = _____ + _____



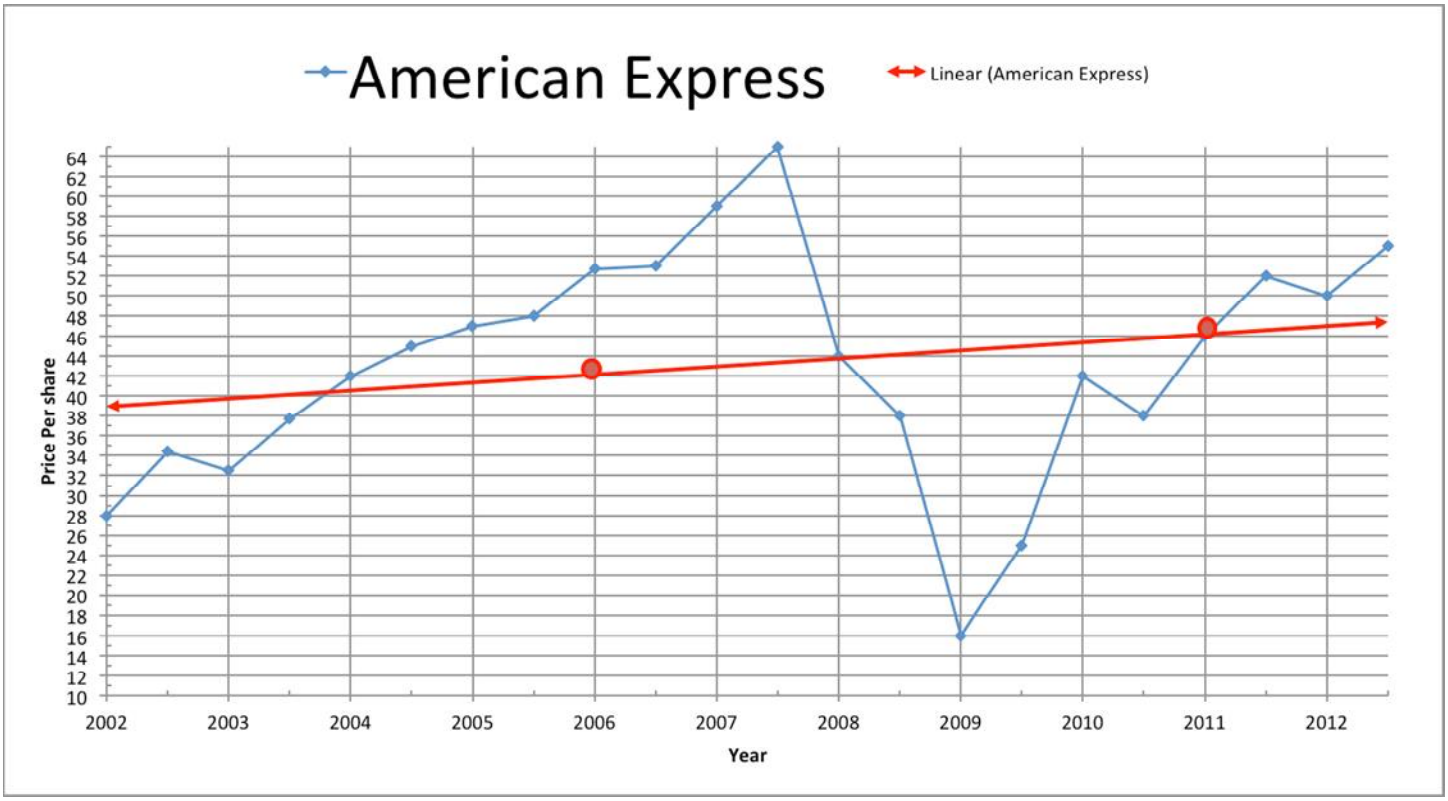
RATE OF CHANGE: PRICE IN 2026:

FUNCTION FOR STOCK GROWTH: PERCENT CHANGE:



RATE OF CHANGE: PRICE IN 2026:

FUNCTION FOR STOCK GROWTH: PERCENT CHANGE:



RATE OF CHANGE: PRICE IN 2026:

FUNCTION FOR STOCK GROWTH: PERCENT CHANGE:

Check with the teacher to make sure you're doing these correctly. He has the answers to these algorithms.

Now it's your turn to pick a stock! Make a five year graph like above and predict the growth of a stock of your choice.
