## $8^{\text {th }}$ grade <br> Black Belt Training: Functions Discipline

Order of Mastery: SYSTEMS
(8.SP.4),(8.EE.8)

1. Graphing
2. Solving methods
3. Supply and Demand

Welcome to the Black Belt - Systems! More specifically, systems of linear equations. A system simply means more than one. So if you graph two different lines that would be graphing a system. But that's not the whole point of this belt. While it's important to know how to graph and solve these types of equations, it's more important to understand what they're used for. How much should McDonald's charge for a hamburger? They don't come up with a price randomly. The cheaper the burger, the more people will buy it. But when do you start losing money? Understanding systems help McDonald's make millions of more dollars each year. So let's figure out how to make more money!

Good Luck Grasshopper.

## Standards Included:

CCSS.Math.Content.8.SP.A. 4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

CCSS.Math.Content.8.EE.C.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

CCSS.Math.Content.8.EE.C.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3 x+$ $2 y=5$ and $3 x+2 y=6$ have no solution because $3 x+2 y$ cannot simultaneously be 5 and 6 .

CCSS.Math.Content.8.EE.C.8c Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

## Criss-Cross

So what exactly is a system of linear equations? It's actually a lot simpler than you may think. A system is merely more than one line on a graph at the same time. When you're asked to SOLVE a system of linear equations, what they're really asking is where the two lines CROSS. Where do they intersect? To find that out, the first way that comes to mind is to just graph each line separately on a piece of graph paper and find that one point where they cross. At that point, $x$ will have a value, and so will $y$.


In the example above, the point at which they cross is $(2,4)$. That means the x value in this system equals 2 and the $y$ value equals 4 . If you were to plug those numbers into the equations of both lines, they will both represent solutions.

The equation of line $A$ above is $y=-1 / 2 x+5$
The equation of line $B$ above is $y=1 / 2 x+3$
These two lines have exactly one thing in common. The point $(2,4)$ lies on both lines. That is what we're trying to find out in a system of linear equations.

To check, just plug in your answer into each equation to see if it works:

$$
\begin{aligned}
4 & =-1 / 2(2)+5 \\
4 & =-1+5 \\
4 & =4
\end{aligned}
$$

AND

$$
\begin{aligned}
4 & =1 / 2(2)+3 \\
4 & =1+3 \\
4 & =4
\end{aligned}
$$

Does this point work in both equations? Check each one to see. Answer yes or no. Show all work.

$$
\text { 1. } \begin{align*}
3 x-2 y & =11  \tag{-2,1}\\
-x+6 y & =7 \tag{5,2}
\end{align*}
$$

2. $6 x-3 y=-15$
$2 x+y=-3$
3. $x+3 y=15$ $4 x+y=6$
4. $-2 x+y=11$
$-x-9 y=-15 \quad(6,1)$

Graph each line ON A SEPARATE SHEET OF GRAPH PAPER. Then write down where they intersect. Then check the solution algebraically
8. $y=-x+3$
9. $y=-6$
$y=x+1$
$x=6$
10. $y=2 x-4$
$2 y=-x$
11. $2 x-3 y=9$
$x=-3$
12. $5 x+4 y=16$ $y=-16$
13. $x-y=1$
$5 x-4 y=0$

## CHALLENGE PROBLEM:

Two lines cross at the point, $(-4,-2)$ and are perpendicular to each other. One of the lines also passes through the point, $(-2,1)$. What are the equations of the two lines?

## The Substitute

A system of linear equations is simply two lines that cross each other at some specific point. So when you're solving a system, you're essentially solving for the point where the two lines cross. It's the one point that when plugged in, should work for BOTH of the equations. But as you probably noticed in last night's homework, graphing the two lines to find the answer doesn't always work. Sometimes they intersect at a fraction. Plus, we could really use a more efficient way to solve it since graphing both lines takes a long time. Enter THE SUBSTITUTION METHOD.

Example: Solve the linear system

$$
\begin{gathered}
-x+y=1 \\
2 x+y=-2
\end{gathered}
$$

Step 1: Solve one of the equations for a variable (pick the easiest path)
In this case, the easiest variable to solve for would be $y$ in the first equation:

$$
\begin{aligned}
& -x+y=1 \\
& \underline{+x} y=\underline{x+1} \\
& y+1
\end{aligned} \quad \text { Add } x \text { to both sides to isolate the variable }
$$

Step 2: Substitute your answer for step 1 into the $2^{\text {nd }}$ equation
Wherever there's a $y$ in the second equation, replace it with your solution in step $1: x+1$

$$
\begin{aligned}
& 2 x+y=-2 \\
& 2 x+(x+1)=-2
\end{aligned}
$$

Step 3: Solve for the remaining variable
In this case, we're solving for x :

$$
\begin{array}{cl}
2 \mathrm{x}+\mathrm{x}+1=-2 & \\
3 \mathrm{x}+1=-2 & \text { combine like terms } \\
\frac{-1}{-1}=\frac{-1}{3} & \text { isolate the variable by subtracting the } 1 \\
\frac{-3}{3} & \text { isolate variable by dividing by } 3 \\
\mathrm{x}=-1 &
\end{array}
$$

Step 4: Substitute your answer back into the revised equation in step 1 to find the other variable.

Since we solved for x , wherever there's an x in step 1 , put in a -1

$$
\begin{aligned}
& y=x+1 \\
& y=-1+1 \\
& y=0
\end{aligned}
$$

So the solution is $x=-1$ and $y=0$, which is written as: $\quad(\mathbf{- 1}, \mathbf{0})$
Critical Thinking: DO NOT SOLVE THESE SYSTEMS!! Just tell me which variable you would solve for first in each system. Which variable would be the easiest? Circle your answer.

1. $2 \mathrm{x}+\mathrm{y}=-10$
2. $m+4 n=30$
$3 x-y=0$
$m-2 n=0$
3. $5 \mathrm{c}+3 \mathrm{~d}=11$
$5 \mathrm{c}-\mathrm{d}=5$
4. $3 x-2 y=19$
$x+y=8$
5. $4 a+3 b=-5$
$a-b=-3$
6. $3 x+5 y=25$
$x-2 y=-10$

Use the substitution method to solve the following systems. SHOW ALL WORK!!!!!
7. $y=x-4$
$4 x+y=26$
8. $x=y+4$
$2 y+x=19$
9. $2 \mathrm{c}-\mathrm{d}=-2$
$4 c+d=20$
10. $2 \mathrm{a}=8$
$a+b=2$
11. $2 x+3 y=31$
$y=x+7$
12. $\mathrm{p}+\mathrm{r}=4$
$4 \mathrm{p}+\mathrm{r}=1$
13. $x-2 y=-25$
$3 x-y=0$
14. $u-z=0$
$7 \mathrm{u}+\mathrm{z}=0$
15. $x-y=0$
$12 x-5 y=-21$

## Challenge:

1. One share of Disney's stock is worth three times as much as a share of Kmart's stock. You invested 100 shares of each. If the total value of the stocks is $\$ 4500$, how much money did you invest in each stock?
2. The admission fee at a small fair is $\mathbf{\$ 1 . 5 0}$ for children and $\$ 4.00$ for adults. On a certain day, 2200 people enter the fair and $\mathbf{\$ 5 0 5 0}$ is collected. How many children and how many adults attended?

## What's the Combination?

So we tried graphing them first... and that was a bit of a pain. Then we used substitution, which can be helpful at times, but also a pain. But not to worry....we have yet another way to solve linear systems. It's called linear combinations. And it's probably the easiest. (Thanks for saving it for last!!!)

Example: Combine the equations to find the solution to the system.

$$
\begin{aligned}
& x+y=4 \\
& x-y=-10
\end{aligned}
$$

Step 1: Arrange the equations in the same order, with like terms in columns.
Woohooo!!! They're already lined up, so we don't have to do anything here:
They both have the $x$ first, then the $y$, and equal to a number

$$
\begin{aligned}
& x+y=4 \\
& x-y=-10
\end{aligned}
$$

Step 2: Multiply, if necessary, to get coefficients that are opposite for one of the variables. In our case, we can multiply the top equation by -1 to get the opposite:

$$
\begin{aligned}
& x+y=4 \\
& -1(x+y=4) \quad \text { Multiply by }-1 \\
& -x-y=-4 \\
& \text { Now our equations look like this: } \\
&
\end{aligned} \begin{aligned}
& -x-y=-4 \\
& \\
& \\
& \\
& \\
& -y-y=-10
\end{aligned}
$$

Step 3: Add 'em up. Line the equations on top of each other and add the columns of like terms. This will eliminate one variable, so you can solve for the other.

$$
\begin{aligned}
&-x-y=-4 \\
& \frac{x-y}{-x-10} \\
& \frac{-2 y}{-2}=\frac{-14}{-2} \\
& y=7
\end{aligned} \quad \text { the x's cancel out } \quad \text { divide both sides by }-2
$$

Step 4: Substitute that answer back into one of the original equations to solve for the other variable.

$$
\begin{aligned}
& x+y=4 \\
& x+(7)=4 \\
& \quad-7 \quad-7 \\
& x=-3
\end{aligned}
$$

So the solution is $\mathbf{x}=-3$ and $\mathrm{y}=7$, which is written as $(-3,7)$

Add 'em up! Combine the equations to find the solution to the system

1. $\mathrm{a}-\mathrm{b}=8$
$a+b=20$
2. $2 x+y=4$
$x-y=2$
3. $m+3 n=2$
$-m+2 n=3$

Multiply....then add 'em up. Combine the equations to find the solution to the system
4. $x+y=8$
$3 x-2 y=19$
5. $4 a+3 b=-5$
$a-b=-3$
6. $2 \mathrm{~g}-3 \mathrm{~h}=0$
$3 g-2 h=5$
7. $5 \mathrm{e}+4 \mathrm{f}=9$
$4 e+5 f=9$

Re-arrange....then figure out the solution
8. $x-3 y=30$
$3 y+x=12$
9. $3 \mathrm{~b}+2 \mathrm{c}=46$
$5 \mathrm{c}+\mathrm{b}=11$

Your choice! Use graphing, or substitution, or addition, subtraction, multiplication....whatever it takes!
10. $\mathrm{y}=\mathrm{x}-9$
$x+8 y=0$
11. $\mathrm{m}=3 \mathrm{n}$
$m+10 n=13$
12. $2 \mathrm{z}=150-\mathrm{u}$
$2 u=150-z$
13. $x+3 y=3$
$x+6 y=3$
14. $v-w=-5$
$v+2 w=4$
15. $5 \mathrm{~s}+8 \mathrm{v}=70$
$60=5 \mathrm{~s}-8 \mathrm{v}$

## Challenge (extra credit)

Your Math test is worth 100 points and has 38 problems. Each problem is worth either 5 points, or 2 points. How many problems of each point value are on the test?

## Wheel of Fortune



Each space has a number on it. If you get an answer that has that number, place the letter of that answer above that number. The goal is to find out what the person's name is, while secretly learning how to solve a system of linear equations at the same time. Not every answer will belong in the puzzle, so you need to find out the value of each letter. Show your work for every problem to get credit. Solve 'em however you want!

1. $5 \mathrm{~h}-\mathrm{s}=-53$
$5 h+3 s=-41$
2. $s+3 y=21$
$2 s-y=0$
3. $\mathrm{u}-\mathrm{e}=4$
$u=-2 e-11$
4. $r=-5 y+41$
$r-4 y=-40$
5. $-4 x-10 c=-4$
$\frac{3}{2} x+5 c=-1$
6. $2 \mathrm{a}+\mathrm{b}=8$
$-4 a+\frac{5}{4} b=-42$
7. $2 L-3 v=4$
$-\frac{4}{5} \mathrm{~L}+\mathrm{v}=-\frac{8}{5}$
8. $\frac{3}{2} \mathrm{z}+\frac{1}{2} \mathrm{c}=\frac{1}{2}$

$$
-3 z+3 c=-9
$$

## Games R’ Us

## BURGERRAMA EXAMPLE:

Harry Potter is marketing director for the BurgerRama restaurant chain. BurgerRama has decided to have a cartoon-character doll made to sell at a premium price at participating BurgerRama locations. The company can choose from several different versions of the doll that sell at different prices. Potter's problem is to decide which selling price will best suit the needs of BurgerRama's customers and store managers. Potter has data from previous similar promotions to help him make a decision.

| Selling Price of <br> Each Doll | Number Supplied <br> per Week per Store | Number Requested <br> per Week per Store |
| :---: | :---: | :---: |
| $\$ 1.00$ | 35 | 530 |
| $\$ 2.00$ | 130 | 400 |
| $\$ 4.00$ | 320 | 140 |



- If Potter sets the price at $\$ 2.50$ per doll, how many disappointed customers would there be per week?
- If Potter sets the price at $\$ 3.80$ per doll, how many unsold dolls will remain at each store per week?
- If the company could give the dolls away, how many would each store need per week?
- According to this graph, what price would make the doll supply so tight that the average number available to each store would be zero?
- Estimate the price where supply and demand will be in equilibrium.


## Games R' Us

After working as the janitor for over a year, the boss of Games R' Us has given you a promotion. He noticed how smart you looked with all your Algebra work, so he decided to make you the marketing manager. The first task: Figure out what price you should make their brand new computer game called Mana-mana doo-doooo-di-doo-doo. To get the maximum profit you need to make sure that supply and demand meet. With your knowledge of systems of linear equations, you know exactly what to do. Your boss gave you some data from previous promotions to help you make your decision. Good luck! If you blow it, you'll be back cleaning out toilets and mopping the floor.

| Price | Supply | Demand |
| :---: | :---: | :---: |
| $\$ 20$ | 150 | 500 |
| $\$ 30$ | 250 | 400 |
| $\$ 50$ | 450 | 200 |

1. You'll need to set up a graph for this data on a separate sheet of graph paper.
2. Plot points representing the selling price and supply on your graph, draw a line through these points and label it Supply
3. Plot points representing the selling price and demand on your graph, draw a line through these points and label it Demand.
4. If you set the price at $\$ 27$ per game, how many disappointed customers will each store have during the week?
5. If you set the price at $\$ 38$ per game, how many unsold games will remain at each store at the end of a week?
6. According to this graph, if the company could give the games away, how many would each store need per week?
7. Estimate the price where supply and demand will be in equilibrium.
8. Make an equation for the supply line.
9. Make an equation for the demand line.
10. Solve the system of linear equations.
11. Does your answer here agree with your answer in question 8 ?

Challenge:Anthony likes to buy and sell coins at the flea market on weekends. He is especially interested in Susan B. Anthony silver dollars. By his own trial-and-error experiences and by information gained from other traders, Anthony has found the following data: Find the supply and demand equations, and the equilibrium.

| Selling <br> Price | Number in <br> Supply | Number in <br> Demand |
| :---: | :---: | :---: |
| $\$ 1.40$ | 10 | 90 |
| $\$ 2.00$ | 19 | 50 |
| $\$ 3.00$ | 42 | 25 |
| $\$ 4.20$ | 94 | 20 |

## Crack the Code <br> (deciphering systems)

Let's go over a couple of word problems that involve systems of linear equations. The key to cracking them is to get the equations down on the page so you don't have to read the problem anymore. Then it's just a matter of solving an equation. As you read a word problem think of these things:

Given: What information did they give you? Write down everything they gave.
Want: What are they asking for? That's the specific question they want answered.
Need: What do you need to know in order to find that answer?

1. Rishika has a total of twenty-five dimes and pennies. She has eleven more pennies than dimes. How many of each coin does she have?

Given:

Want:

Need:
2. Aowei has twelve fewer dimes than nickels. He has a total of $\$ 2.10$. How many of each coin does he have?

Given:
Want:
Need:

You're on your own! That means you can't ask the teacher! You have to use your code cracking skills to try and figure these out. Make sure you write down the three things to help in your journey.
3. The sum of two numbers is 158 . Two times the smaller number plus twenty-nine is the larger number. What are the numbers?
4. Kevin has thirteen fewer pennies than dimes. Kevin has a total of forty-seven pennies and dimes. How many of each coin does he have?
5. Michael has a total of eighteen nickels and dimes. The total value of the coins is $\$ 1.30$. How many of each coin does he have?
6. D.J. has three more dimes than nickels. He has a total of forty-seven nickels and dimes. At the mall, D.J. spent eleven nickels and four dimes. How much money does he have left?
7. If Ashley had eleven fewer pennies, she would have three times as many pennies as dimes. The total value of the coins is $\$ 1.54$. How many of each coin does she have?

## Practice those systems!!!

8. $x+5 y=-43$
$5 x+20 y=-170$
9. $x+3 y=8$
$5 x-6 y=-40$
10. $-3 x+3 y=39$
$3 x+9 y=81$
11. $-3 x-y=24$
$-3 x+3 y=0$
12. One number is thirty-six more than three times another number. If each number were multiplied by five, their difference would be 400 . What are the numbers?
13. The larger of two numbers is five more than three times the smaller number. The larger number is also forty-four more than two times the smaller number. What are the numbers?
14. Isaac is five times as old as Jordan. Madison is twice as old as Jordan. Jordan is fourteen years younger than Madison. How old is Jordan?
15. Daniel is seven years younger than Jacob. Luis is eight years younger than Daniel. Jacob is fifteen years older than Luis. The sum of all three ages is ten more than four times the age of Luis. How old is Daniel?
16. Kaitlyn is sixteen years older than Matthew. Kaitlyn is twelve years older than Isaac. Isaac is four years older than Matthew. The sum of their ages is sixty-two. How old is Matthew?

# Numbers, Ages and Coins 

## These are probably the most common problems associated with systems. They can all be figured out via the methods we've learned. What two equations can you write? What are the unknowns? Remember your code cracking skills!

## Let's start with the number riddles

1. A number is nine more than five times another number. Their sum is one hundred ninety-five. What are the numbers?
2. A number is fourteen more than four times another number. Their difference is one hundred four. What are the numbers?
3. The sum of two numbers is sixty-five. Their difference is forty-one. What are the numbers?
4. A number is two times as large as another number. Five times the smaller number is sixty more than the smaller number. What are the numbers?
5. The sum of two numbers is 70 . Two times the smaller number plus nineteen is the larger number. What are the numbers?
6. One number is ninety less than seven times another number. If each number were multiplied by six, their difference would be 252 . What are the numbers?
7. The larger of two numbers is seven more than five times the smaller number. The larger number is also one hundred thirty-six more than two times the smaller number. What are the numbers?
8. Three times the sum of two numbers is 51 . Five times the sum of the same two numbers is 85 . The difference of the two numbers is 5 . What are the numbers?

## AGE

9. Austin is nine years older than Daniel, who is four years older than Jordan. The sum of their ages is fifty-six. How old is Daniel?
10. Jason is ten and one-half years older than James, who is five and five-sixth years older than Emma. The sum of their ages is 57 and eleven-twelfth years. How old is Emma?
11. Julia is five years younger than William. William is three times as old as Matthew. The sum of their ages is seventy-nine. How old is Matthew?
12. Eric is thirty less than four times the age of Thomas. Hailey is twenty-eight less than three times the age of Thomas. The sum of their ages is sixty-two. How old is Hailey?
13. Benjamin is five years older than Alexandra. Kaylee is three years younger than Alexandra. Benjamin is eight years older than Kaylee. The sum of their ages is sixty-five. How old is Kaylee?
14. Jasmine is four years older than Anthony. Rebecca is nine years older than Anthony. Jasmine is five years younger than Rebecca. The sum of all three ages is seven more than two times the age of Rebecca. How old is Jasmine?
15. Makayla is four times as old as Lauren. Tyler is twice as old as Lauren. Makayla is thirty-two years older than Tyler. How old is Tyler?
16. Katherine is two years younger than Elizabeth. Katherine is twice as old as Taylor was five years ago. Taylor and Katherine are the same age. How old is Taylor?

## COINS

17. Alexander has a total of thirty-three pennies and quarters. He has thirteen more quarters than pennies. How much money does he have?
18. Joshua has twelve fewer pennies than quarters. Joshua has a total of thirty-four pennies and quarters. How much money does he have?
19. Jasmine has sixteen more quarters than nickels. Jasmine has a total of $\$ 11.20$. How many of each coin does she have?
20. Kayla has a total of seventeen dimes and quarters. The total value of the coins is $\$ 2.75$. How many of each coin does she have?
21. Samantha has three times as many dimes as nickels. The total value of the coins is $\$ 3.15$. How many of each coin does she have?
22. If Thomas had fifteen fewer quarters, he would have four times as many quarters as dimes. The total value of the coins is $\$ 20.25$. How many of each coin does he have?
23. Ethan has four times as many dimes as pennies. If Ethan had thirty-eight more pennies and twenty-two fewer dimes, he would have the same number of each coin. How much money does he have?
24. Alyssa has $\$ 10.70$ in nickels and dimes. Alyssa has seven more dimes than nickels. How many of each coin does she have?

## Systems of linear inequalities

We know how to graph an inequality like: $y>2 x+2$. We find the boundary line by making it an equation, and then shade the appropriate side. Well now we're going to graph a system of inequalities.

Example: Graph the system of inequalities: $\quad \mathbf{x}+\mathbf{y}<\mathbf{3}$
$x+4 y \geq 0$
Step 1: Graph the boundary lines of each inequality. (To do that, we made them equations, and graphed the lines. Remember to use a dashed line for a $>$ or $<$, use a solid line if $\geq \mathrm{or} \leq$ )


$$
x+y=3 \quad x \quad y
$$

$$
x+4 y=0
$$



Step 2: Shade the appropriate side for each inequality. Test a point for each to see.


Step 3: Shade in the overlap of the two regions a little darker than the rest. This is where they intersect. This is your answer! You can check it by testing a point in the overlap in both inequalities.

Graph the following systems of inequalities: SHOW ALL WORK ON ANOTHER SHEET OF PAPER!

1. $\mathrm{y}>0$
$x \geq-2$

2. $y>-2$
$y \leq 4-2 x$

3. $2 x+3 y<5$
$3 x+2 y>5$

4. $y<2 x-1$

5. $2 x-2 y \leq 6$

6. $x-3 y \geq 12$
$x-6 y<12$

7. $x>0$
$\mathbf{y}>0$
$\mathbf{x}<3$

8. $x<3$
$2 y<1$
$2 x+y>2$
9. $x-2 y<3$
$3 x+2 y>9$
$x+y<6$

Challenge: You can work a total of no more than 20 hours a week at your two jobs. Baby-sitting pays $\$ 5$ an hour, and your job as a cashier pays $\$ 6$ an hour. You need to earn $\$ 90$ a week to cover your expenses. Write a system of linear inequalities that shows the various numbers of hours you can work at each job?

WHAT WENT WRONG?

It's your turn to grade! This is my test. Not every problem is wrong. But since you're grading, you need to check every one. If it's correct, give it a star. If it's wrong, then on A SEPARATE SHEET OF PAPER, write what I did wrong, and rework it. This is your ticket to taking the belt test.
1.

$$
\begin{gathered}
\begin{array}{l}
3 x-3 y=-6 \\
x-4 y=-11
\end{array} \\
y-2-4 y=- \\
+2 \\
\frac{-3 y}{-3}=\frac{9}{-3} \\
y=-3 \\
x=-5 \\
y=-3
\end{gathered}
$$

$$
\begin{gathered}
3 x-3^{3} y=-6^{+3 y} \\
\frac{3 x}{3}=\frac{3 y-6}{3} \\
x=y-2 \\
3 x-3(-3)=-6 \\
3 x+q=-6 \\
-q \\
\frac{3 x}{3}=-\frac{15}{3} \\
x=-5
\end{gathered}
$$

$$
\begin{aligned}
& x-4 y=-11 \\
& y-2-4 y=-11 \\
&+2 \frac{3 x}{3}
\end{aligned} \begin{array}{rl}
x-2 & 3 y-6 \\
3 & =y-2
\end{array}
$$

2. $x+y=20$
$x-y=8 \quad x+y=20$
$-x$
火-20-x=-------------->

$$
-x
$$

$$
y=20-x
$$

$-20=8$
no solution

$$
\begin{gathered}
2(5 x-10)+x=13 \\
10 x-20+x=13 \\
11 x-20=13 \\
+20+20 \\
11 x=33
\end{gathered}
$$

$$
\begin{aligned}
& \text { 3. } 2 y+x=13 \\
& y=5 x-10 \\
& y=5(3)-10 \\
& y=15-16 y=5
\end{aligned}
$$

4. Write an inequality


$$
y<\frac{1}{3} x+\frac{2}{3}
$$

$$
\begin{aligned}
& m=\frac{1}{3} \\
& y=m x+b \\
& y=\frac{1}{3} x+b \\
& z=\frac{1}{3}(4)+b \\
& 2=\frac{4}{3}+b \\
& -\frac{4}{7}-\frac{4}{3}
\end{aligned}
$$

5. Your egg drop project was dropped from 30 feet. It took just 1.5 seconds to hit the ground. How long would it take, in seconds to hit the ground if you dropped it from 250 feet? Convert that to minutes.

$$
\frac{30}{1.5} \cdot \frac{250}{1}=5000 \text { ELS } 83 \mathrm{~min}
$$

6. 

| Selling Price of <br> Each Doll | Number Supplied <br> (in thousands) | Number Requested <br> (in thousands) |
| :---: | :---: | :---: |
| $\$ 1$ | 35 | 530 |
| $\$ 2$ | 130 | 400 |
| $\$ 4$ | 320 | 140 |

This is the data for past sales of erasers at Erasers R' Us. Your job is to figure out what price you should sell the company's brand new eraser, The Uber-Eraser 5000.
a. Write an equation of the supply line based on the data.

$$
\begin{aligned}
& \text { s to figure out what price you should sell } \\
& \frac{130-35}{2-1}=\frac{95}{1}=m=95
\end{aligned}
$$

$$
\left(\begin{array}{l}
2-1 \\
y=95 x-60 \\
(1,35) \\
y=m x+b \\
25=95(1)+
\end{array}\right.
$$

b. Write an equation of the demand line based on the data.
c. If you gave away the erasers, how many people would want them according to youndata?

$$
=660
$$

$$
y=130 x-660
$$

$$
\begin{aligned}
& \text { d. What price should youmake the eraser? }
\end{aligned}
$$

## ANSWERS TO GRAPHING DISCIPLINE BLACK BELT

## CRISS CROSS



## THE SUBSTITUTE

1. $2 x+y=-10$
2. $\begin{aligned} \text { (m) }+4 \mathrm{n} & =30 \\ \mathrm{~m})-2 \mathrm{n} & =0\end{aligned}$
3. $5 \mathrm{c}+3 \mathrm{~d}=11$
$3 x-y=0$
$5 \mathrm{c}-\mathrm{d}=5$
4. $3 x-2 y=19$
(8)
5. $4 a+3 b=-5$
(a) $-b=-3$
6. $3 x+5 y=25$
( $\mathbf{x}-2 \mathrm{y}=-10$
7. $(6,2) \quad$ 8. $(9,5) \quad$ 9. $(3,8) \quad$ 10. $(4,-2) \quad$ 11. $(2,9) \quad 12 .(-1,5) \quad 13 .(5,15) \quad 14 .(0,0) \quad 15 .(-3,-3)$

CHALLENGE: Disney \$3375 Kmart \$1125

## WHAT'S THE COMBINATION?

1. $(14,6)$ 2. $(2,0)$ 3. $(-1,1)$ 4. $(7,1) \quad$ 5. $(-2,1) \quad 6 .(3,2) \quad 7 .(1,1) \quad 8 .(21,-3) \quad$ 9. $(16,-1)$ 10. $(8,-1)$ 11. $(3,1)$ 12. $(50,50)$ 13. $(3,0) \quad 14 .(3,-2) \quad 15 .(13,5 / 8)$

CHALLENGE: 30 ( 2 pointers) 8 ( 5 pointers)

## GAMES R US

Answers
Sheet 1: Burgerama
Students' graphs should look like the one in figure 1.


Fig. 1

Application 1:Answers to $1 \mathrm{a}-1 \mathrm{e}$ will vary slightly.
1a) about 170 customers 1b) about 140 customers 1c) about 660 dolls 1 d ) about $\$ 0.65$ per doll $\begin{array}{lll}\text { 1e) about } \$ 3.20 & 2 a) S=95 P-60 & 2 b) D=-130 P+660(P \text { is in dollars) } \quad 2 c) \$ 3.20\end{array}$

Answers to Games R' Us.
Questions 1-3 are graphed 4) 210 customers disappointed $\quad$ 5) 10 unsold games $\quad$ 6) $700 \quad$ 7) $\$ 5.00$
8) Around $\$ 37-38$
9) Supply:
or $S=10 P-50$
10)Demand
$y=-10 x+700$ or
D $=-10 P+700$
11) $\$ 37.50$
12) yes

## CHALLENGE

Students' graphs should be similar to the one in figure 2.


Fig. 2

The equilibrium is about $\$ 2.76$ The equation for the supply: $y=30 x-36$
The equation for the demand: $\quad y=-20 x+102$

## CRACK THE CODE

1. 18 pennies 7 dimes $\quad 2.22$ nickels 10 dimes $\quad 3.43$ and $115 \quad 4.17$ pennies 30 dimes
2. 10 nickels 8 dimes 6 . $\$ 2.6511$ nickels 21 dimes 7.44 pennies 11 dimes
3. $x=2 y=-9$
4. $x=-8 \quad y=0$
5. $x=-3 \quad y=10 \quad$ 11. $x=-6 \quad y=-6 \quad$ 12. 22 and 102
6. 39 and 122
7. 14 15. 21
8. 14

## NUMBERS AGES AND COINS

$\left.\begin{array}{lllllll}\text { 1. } 31 & \text { and } 164 & \text { 2. } 30\end{array}\right)$ and $134 \quad$ 3. 12 and $53 ~ 4 . ~ 15$ and $30 \quad$ 5. 17 and $53 \quad 6.22$ and 647.43 and 222
8. 6 and $11 \quad 9.17 \quad 10.11$ and eleven-twelfth years $11.12 \quad 12.17 \quad 13.18$ 14.16 $15.32 \quad 16.10$ 17. $\$ 5.85$. ( 10 pennies 23 quarters) 18. $\$ 5.86$. (11 pennies 23 quarters) 19. 24 nickels 40 quarters
20. 10 dimes 7 quarters 21. 9 nickels 27 dimes 22. 15 dimes 75 quarters $\quad 23$. $\$ 8.20 .20$ pennies 80 dimes
24. 76 nickels and 69 dimes

## SYSTEMS OF LINEAR INEQUALITIES


2. $\mathrm{y}>-2$
7. $x>0$
y>0

5. $2 x-2 y \leq 6$

8. $\mathrm{x}<3$
$2 y<1$

3. $2 \mathrm{x}+3 \mathrm{y}<5$

6. $\mathrm{x}-3 \mathrm{y} \geq 12$
$x-6 y<12$

9. $\mathrm{x}-2 \mathrm{y}<3$
$3 x+2 y>9$
$x+y<6$

Challenge: You can work a total of no more than 20 hours a week at your two jobs. Baby-sitting pays $\$ 5$ an hour, and your job as a cashier pays $\$ 6$ an hour. You need to earn $\$ 90$ a week to cover your expenses. Write a system of linear inequalities that shows the various numbers of hours you can work at each job. $\mathrm{b}+\mathrm{c} \leq \mathbf{2 0} \quad \mathbf{5 b}+\mathbf{6 c} \geq \mathbf{9 0}$

